Strang: Chapter 5

Section 5.1. Exercises 1, 3, 8, 14, 21, 27, 33

Section 5.2. Exercises 2, 5, 7, 11, 12, 16, 20, 23, 32

5.2-2: Use exercise 1 in part (a). **5.2-12**: Find the cofactor matrix for A. Then compare the inverse of A with AC^T.

Problem week10-1. The first three Legendre polynomials are 1, x, and $x^2 - 1$. Choose c so that the fourth polynomial $x^3 - cx$ is orthogonal to the first three. All integrals go from -1 to 1. See Exercise 8.5-4.

Problem week10-2. Graph the square wave. Then graph by hand the sum of two sine terms in its series, or graph by machine the sum of 2, 3, and 10 terms. The famous Gibbs phenomenon is the oscillation that overshoots the jump (this doesn't die down with more terms). See Exercise 8.5-7.

Section 5.3. Exercises 2, 4, 6, 9, 17, 28, 33, 36

Some Answers

5.1. Exercises 1, 8, 14, 21, 27 have textbook answers.

5.1-3. (a) False: $\det(I+I)$ is not $1+1$ (b) True: The product rule extends to ABC (use it twice) (c) False: $\det(4A)$ is $4^n \det(A)$ (d) False: $A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $AB - BA = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is invertible.

5.1-33. I now know that maximizing the determinant for $1, -1$ matrices is Hadamard's problem (1893): see Brenner in American Math. Monthly volume 79 (1972) 626-630. Neil Sloane's wonderful On-Line Encyclopedia of Integer Sequences (`research.att.com/~njas`) includes the solution for small n (and more references) when the problem is changed to 0, 1 matrices. That sequence `A003432` starts from $n = 0$ with $1, 1, 1, 2, 3, 5, 9$. Then the $1, -1$ maximum for size n is 2^{n-1} times the 0, 1 maximum for size $n-1$ (so $(32)(5) = 160$ for $n = 6$ in sequence `A003433`). To reduce the $1, -1$ problem from 6 by 6 to the 0, 1 problem for 5 by 5, multiply the six rows by ± 1 to put $+1$ in column 1. Then subtract row 1 from rows 2 to 6 to get a 5 by 5 submatrix S of -2, 0 and divide S by -2. Here is an advanced MATLAB code and a $1, -1$ matrix with largest $\det(A) = 48$ for $n = 5$:

```
 n=5; p=(n-1)^2; A0=ones(n); maxdet=0;
for k=0 : 2^p - 1
 Asub=rem(floor(k. * 2.^(-p + 1: 0)),2);
 A=A0;
 A(2:n,2:n)= 1-2*reshape(Asub,n-1,n-1);
 if abs(det(A))>maxdet, maxdet=abs(det(A)); maxA=A;
 end
end
```

$$\text{Output:} \qquad maxA = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 & 1 \end{bmatrix}, \quad maxdet = 48$$

5.2. Exercises 2, 11, 12, 16, 20, 32 have textbook answers.

5.2-5. Four zeros in the same row guarantee $\det = 0$. $A = I$ has 12 zeros (maximum with $\det \neq 0$).

5.2-7. $5!/2 = 60$ permutation matrices have det $= +1$. Move row 5 of I to the top; starting from rows in the order $(5, 1, 2, 3, 4)$, elimination to reach I will take four row exchanges.

5.2-23.

(a) If we choose an entry from B we must choose an entry from the zero block; result zero. This leaves entries from A times entries from D leading to $\det(A)\det(D)$.

(b) and (c) Take $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. See the solution to problem 25.

Problem week10-1. Answer: $c = 3/5$. Use $\int_{-T}^{T}(\text{odd function})dx = 0$ in the report.

Problem week10-2. The -1, 1 odd square wave is $f(x) = x/|x|$ for $0 < |x| < \pi$. Its Fourier series in equation (8), Section 8.5 of Strang's book, is $4/\imath$ times $[\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x \cdots]$. The sum of the first N terms has an interesting shape, close to the square wave except where the wave jumps between -1 and 1. At those jumps, the Fourier sum spikes the wrong way to ± 1.09 (the Gibbs phenomenon) before it takes the jump with the true values of $f(x)$.

This happens for the Fourier sums of all functions with jumps. It makes shock waves hard to compute. You can see it clearly in a graph of the sum of 10 terms.

5.3. Exercises 2, 4, 6, 9, 17, 36 have textbook answers.

5.3-28.Let $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\phi$ and $\vec{u} = (x, y, z)$. Then

$$J = \det[\vec{u}_\rho|\vec{u}_\phi|\vec{u}_\theta] = \begin{pmatrix} \sin\phi\,\cos\theta & \rho\cos\phi\,\sin\theta & -\rho\sin\phi\,\sin\theta \\ \sin\phi\,\sin\theta & \rho\cos\phi\,\sin\theta & \rho\sin\phi\,\cos\theta \\ \cos\phi & -\rho\sin\phi & 0 \end{pmatrix}$$

Expand by cofactors along column 1, then simplify with trig identities, obtaining the answer $= \rho^2\sin\phi$.

5.3-33. Find the components of $\vec{v}\times\vec{w}$ from determinant expansion det $\begin{pmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix}$. These are exactly the cofactors along row one of the determinant det $\begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix}$. Then the determinant equals its cofactor expansion $=$ dot product of $\vec{u}$ and $\vec{v}\times\vec{w}$.