

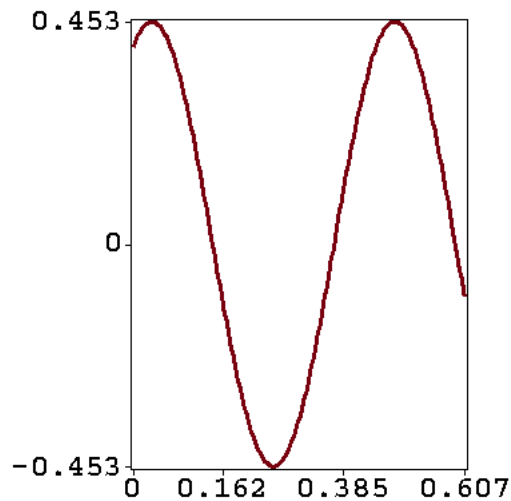
## Quiz 5

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### Quiz 5, Problem 1. Harmonic Vibration

A mass of  $m = 200$  grams attached to a spring of Hooke's constant  $k$  undergoes free undamped vibration. At equilibrium, the spring is stretched 10 cm by a force of 4 Newtons. At time  $t = 0$ , the spring is stretched 0.4 m and the mass is set in motion with initial velocity 3 m/s directed away from equilibrium. Find:

- (a) The numerical value of Hooke's constant  $k$ .
- (b) The initial value problem for vibration  $x(t)$ .
- (c) Show details for solving the initial value problem for  $x(t)$ .  
The answer is  $x(t) = \frac{2}{5} \cos(\sqrt{200}t) + \frac{3}{20} \sqrt{2} \sin(\sqrt{200}t)$ , graphed below.



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**Quiz 5, Problem 2. Harmonic Vibration, Continued**

Assume results (a), (b), (c) from Problem 1. In particular, assume

$$x(t) = \frac{2}{5} \cos(\sqrt{200}t) + \frac{3\sqrt{2}}{20} \sin(\sqrt{200}t).$$

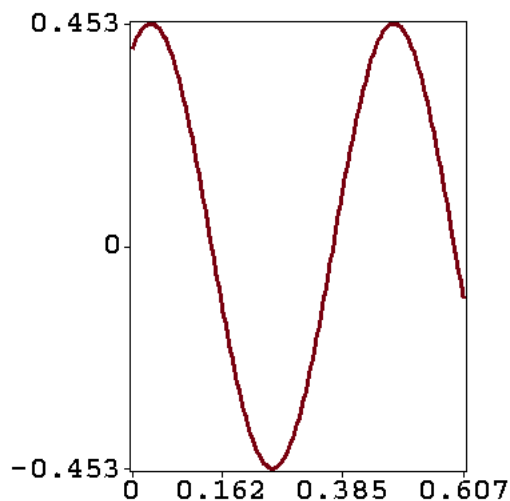
Complete these parts.

(d) Plot the solution  $x(t)$  using technology, approximately matching the graphic below.

(e) Show trig details for conversion of  $x(t)$  to phase-amplitude form

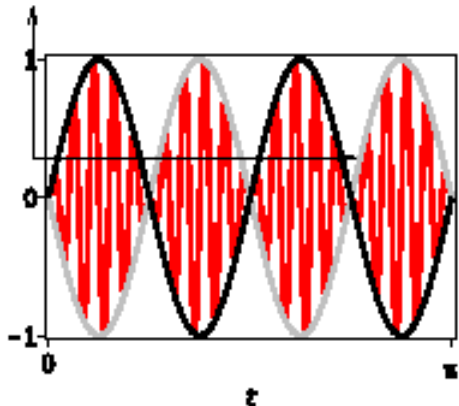
$$x(t) = \frac{\sqrt{82}}{20} \cos(\sqrt{200}t - \arctan(3\sqrt{2}/8)).$$

(f) Report from the answer in part (e) decimal values for the period, amplitude and phase angle. Two-place decimal accuracy is sufficient.



### Quiz 5, Problem 3. Beats

The physical phenomenon of **beats** refers to the periodic interference of two sound waves of slightly different frequencies. A destructive interference occurs during a very brief interval, so our impression is that the sound periodically stops, only briefly, and then starts again with a beat, a section of sound that is instantaneously loud again. An illustration of the graphical meaning appears in the figure below.



#### Beats

Shown in red is a periodic oscillation  $x(t) = 2 \sin 4t \sin 40t$  with rapidly-varying factor  $\sin 40t$  and the two slowly-varying envelope curves  $x_1(t) = 2 \sin 4t$  (black),  $x_2(t) = -2 \sin 4t$  (grey).

The undamped, forced spring-mass problem  $x'' + 1296x = 640 \cos(44t)$ ,  $x(0) = x'(0) = 0$  has by trig identities the solution  $x(t) = \cos(36t) - \cos(44t) = 2 \sin 4t \sin 40t$ .

**The Problem.** Solve the initial value problem

$$x'' + 1444x = 1056 \cos(50t), \quad x(0) = x'(0) = 0$$

by undetermined coefficients and linear algebra, obtaining the solution  $x(t) = \cos(38t) - \cos(50t)$ . Then show the trig details for  $x(t) = 2 \sin(6t) \sin(44t)$ . Finally, graph  $x(t)$  and its slowly varying envelope curves on  $0 \leq t \leq \pi$ .

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