Name _____

Math 2280 Numerical Methods Project
S2017

References: Edwards-Penney, Sections 2.4,2.5, 2.6. This document: numericalDEproject-S2017.pdf

Other related and required documents are available at the course web site, in particular Numerical Methods Slides, Numerical Methods Manuscript, Sample Maple Code for Euler, Heun, RK4.

Symbolic Solution. (E & P Exercises 2.4-6, 2.5-6, 2.6-6 Symbolic Solution)

The symbolic solution of $y' = -2xy$, $y(0) = 2$ is $y = 2e^{-x^2}$. Display the details for the derivation of this symbolic solution, using methods from Edwards-Penney section 1.4 or 1.5. Do a full 2-panel answer check.

The answer $y = 2e^{-x^2}$ can be used to make the following table, which is used in the problems below.

x	0.00	0.10	0.20	0.25	0.30	0.40	0.50
$2e^{-x^2}$	2.000000000	1.980099667	1.921578878	1.878826126	1.827862371	1.704287578	1.557601566

Euler's Method. (E & P Exercise 2.4-6)

Consider the initial value problem $y' = -2xy$, $y(0) = 2$ with symbolic solution $y = 2e^{-x^2}$. Apply Euler's method to produce two dot tables, as shown below. The first has three pairs, $h = 0.25$. The second has six pairs, $h = 0.1$. Reproduce the summary of results below, writing 4 small digits (Geek pen required) into the blank rectangle ☐. The 4th digit can be rounded or not.

The work for $h = 0.25$ is to be entirely hand-written, with calculator assist. Answer checks and the work for $h = 0.1$ may use technology.

Table $h = 0.25$: [0,2], [0.25,2], [0.5,1.75☐]

Table $h = 0.10$: [0,2], [0.1,2.0], [0.2,1.96], [0.3,1.88☐], [0.4,1.76☐], [0.5,1.627☐]

Actual $y(.5)$	Approx $y(.5)$, $h = 0.25$	Approx $y(.5)$, $h = 0.10$
1.557601566	1.75☐	1.627☐

Heun's Method (Modified Euler). (E & P Exercise 2.5-6)

Consider the initial value problem $y' = -2xy$, $y(0) = 2$ with symbolic solution $y = 2e^{-x^2}$. Apply Heun's method (Improved Euler) to produce one dot table of six rows, $h = 0.1$. Reproduce the table below and fill in missing digits. Hand-written work is expected with computer assist. Hand-written work with calculator assist should end after the estimate for $y(0.1)$. Technology should be used for the remaining answers. Four digits are expected in ☐.

x	0.0	0.1	0.2	0.3	0.4	0.5
Actual $y(x)$	2.000000000	1.980099667	1.921578878	1.827862371	1.704287578	1.557601566
Approx $y(x)$	2.0000000	1.980☐	1.921☐	1.827☐	1.704☐	1.557☐

RK4 Method. (E & P Exercise 2.6-6)

Consider the initial value problem $y' = -2xy$, $y(0) = 2$ with symbolic solution $y = 2e^{-x^2}$. Apply the RK4 method to produce one dot table of three rows, $h = 0.25$. Reproduce the table below, filling in the missing digits. Hand-written work should complete the estimate of $y(0.25)$ to 8 digits, using calculator assist. The remaining steps should use technology, with a plan to reproduce the hand-written result. Four digits are expected in ☐.

x	0.00	0.25	0.50
Actual $y(x)$	2.000000000	1.878826126	1.557601566
Approx $y(x)$	2.000000000	1.878☐	1.557☐

Submit this printed page, with blanks filled in, as the first page of your report. Attach hand-written solutions next. Append computer results last, then staple. Assigned in Week 3.