Homework 4 Solutions

Sec. 3.3 (25) $3y^{(3)} + 2y'' = 0 \quad y(0) = -1,\ y'(0) = 0,\ y''(0) = 1$

Char. Equation $3r^3 + 2r^2 = 0 \Rightarrow r^2(3r+2) = 0$

roots: $r = -\frac{2}{3}$, $r = 0$ multiplicity 2. A basis of solutions is

$y_1 = 1,\ y_2 = x,\ y_3 = e^{-\frac{2}{3}x}$, the general solution is

$y = C_1 + C_2 x + C_3 e^{-\frac{2}{3}x}$. Find C_1, C_2, C_3:

$y' = C_2 - \frac{2}{3} C_3 e^{-\frac{2}{3}x}$ sub. $x=0$ into y, y', y''

$y'' = \frac{4}{9} C_3 e^{-\frac{2}{3}x}$

$$\begin{aligned} C_1 \quad + C_3 &= -1 \\ C_2 - \tfrac{2}{3} C_3 &= 0 \\ \tfrac{4}{9} C_3 &= 1 \end{aligned} \Rightarrow C_3 = 9/4,\quad C_2 = \tfrac{2}{3} C_3 = \tfrac{3}{2}$$

$$C_1 = -1 - C_3 = -\tfrac{13}{4}$$

$$y = -\frac{13}{4} + \frac{3}{2} x + \frac{9}{4} e^{-\frac{2}{3}x}$$

(29) $y^{(3)} + 27y = 0$. Characteristic equation is $r^3 + 27 = 0$

$\Rightarrow r^3 = -27 \Rightarrow$ real root $r = -3$. So $r+3$ is a factor of Characteristic Polynomial. Use Long division to get other factors.

$$\begin{array}{r} r^2 - 3r + 9 \\ r+3 \overline{\big) r^3 + 27} \\ r^3 + 3r^2 \\ \hline -3r^2 + 27 \\ -3r^2 - 9r \\ \hline 9r + 27 \\ 9r + 27 \\ \hline 0 \end{array}$$

$\Rightarrow (r+3)(r^2 - 3r + 9) = r^3 + 27$

Other roots: $r^2 - 3r + 9 = 0$

$$\Rightarrow \quad r = \frac{3 \pm \sqrt{9-36}}{2} = \frac{3 \pm \sqrt{-27}}{2} = \frac{3}{2} \pm \frac{\sqrt{27}}{2} i$$

Basis of solutions: $y_1 = e^{-3x}$, $y_2 = e^{\frac{3}{2}x} \cos \frac{\sqrt{27}}{2} x$, $y_3 = e^{\frac{3}{2}x} \sin \frac{\sqrt{27}}{2} x$

General Soln: $\boxed{y = C_1 e^{-3x} + C_2 e^{\frac{3}{2}x} \cos \frac{\sqrt{27}}{2} x + C_3 e^{\frac{3}{2}x} \sin \frac{\sqrt{27}}{2} x}$

(36) $9y^{(3)} + 11y'' + 4y' - 14y = 0$, $y = e^{-x}\sin x$ is a solution,

Therefore $r = -1 \pm i$ must be a characteristic root, and thus $y = e^{-x}\cos x$ is another solution. Since $r = -1 \pm i$ are char. roots, $[r-(-1+i)][r-(-1+i)]$ must be a factor of the characteristic polynomial. This is

$$r^2 - [(-1+i)+(-1-i)]r + (-1+i)(-1-i)$$

$= r^2 + 2r + 2$. We divide into char. polynomial to find other factor

$$\begin{array}{r|l} & 9r - 7 \\ \hline r^2 + 2r + 2 & 9r^3 + 11r^2 + 4r - 14 \\ & 9r^3 + 18r^2 + 18r \\ \hline & -7r^2 - 14r - 14 \\ & -7r^2 - 14r - 14 \\ \hline & 0 \end{array}$$

So $(9r-7)$ is a factor of Char. Polynomial $\Rightarrow r = 7/9$ is a root, so $y = e^{7/9 x}$ also a soln.

Therefore, the general solution is

$$\boxed{y = c_1 e^{\frac{7}{9}x} + c_2 e^{-x}\cos x + c_3 e^{-x}\sin x}$$

Sec. 3.4) 4) s_0 = stretch in spring, then force of 9

stretching it 0.25 m (Newtons are units meters. kg/s^2)

$\Rightarrow \quad F = k s_0 \quad$ so $\quad 9 = k\cdot(0.25) \Rightarrow \boxed{k = 36}$

Mass of object is $250g = 0.25$ kg.

$\Rightarrow \omega_0^2 = \frac{k}{m} = \frac{36}{0.25} = 144 \Rightarrow \omega_0 = 12.$

Thus the general solution is

$$x = c_1 \cos 12t + c_2 \sin 12t$$

$$x' = -12 c_1 \sin t + 12 c_2 \cos t$$

$x(0) = 1 \qquad x'(0) = -5$

$\Rightarrow \quad$ ~~ex~~ $x(0) = c_1 = 1 \quad \Rightarrow \quad c_1 = 1$, $c_2 = -5/12$

$x'(0) = 12 c_2 = -5$

Amplitude $C = \sqrt{c_1^2 + c_2^2} = \sqrt{1 + \frac{25}{144}} = \sqrt{\frac{169}{144}} = \frac{13}{12}$

$\tan\alpha = \frac{c_2}{c_1} = \frac{-5}{12}$, $(1, -\frac{5}{12})$ in 4th quad.

$\Rightarrow \quad \alpha = \tan^{-1}\left(\frac{-5}{12}\right) + 2\pi \cong 5.89$

$$\boxed{x(t) = \frac{13}{12}\cos(12t + 5.89)} \qquad \boxed{\text{Period} = \frac{2\pi}{\omega_0} = \frac{2\pi}{12} = \frac{\pi}{6}}$$

(18) $m=2,\ c=12,\ k=50,\ x_0=0,\ v_0=-8$

$mx''+cx'+kx=0 \Rightarrow 2x''+12x'+50x=0$

Characteristic roots: $\dfrac{-12\pm\sqrt{144-400}}{4} = -3\pm\dfrac{\sqrt{256}i}{4} = -3\pm\frac{16}{4}i$

or $-3\pm 4i$.

General solution: $x = c_1e^{-3t}\cos 4t + c_2e^{-3t}\sin 4t$

$x' = -3c_1e^{-3t}\cos 4t - 4c_1e^{-3t}\sin 4t - 3c_2e^{-3t}\sin 4t + 4c_2e^{-3t}\cos 4t$

$x(0) = c_1 + 0 = 0 \quad\Rightarrow\quad c_1=0 \quad\Rightarrow\quad c_1=0$

$x'(0) = -3c_1+4c_2 = -8 \qquad 4c_2=-8 \qquad c_2=-2.$

$\Rightarrow \boxed{x(t) = -2e^{-3t}\sin 4t}$

$C=\sqrt{c_1^2+c_2^2} = \sqrt{4}=2$

$\tan\varphi = c_2/c_1 = \dfrac{-2}{0} = -\infty$ (interpreted as in 4th quadrant)

so $\varphi = \tan^{-1}(-\infty)+2\pi = -\frac{\pi}{2}+2\pi = \frac{3\pi}{2}.$

$\Rightarrow \boxed{x(t) = 2\cos(4t-\tfrac{3\pi}{2})}$

If $c=0$, $\omega_0=\sqrt{k/m} = \sqrt{\frac{50}{2}} = 5$

$x(t) = c_1\cos 5t + c_2\sin 5t$

$x'(t) = -5c_1\sin 5t + 5c_2\cos 5t$

$x(0) = c_1 = 0$, $x'(0) = 5c_2 = -5 = -8 \quad c_2=-\frac{8}{5}$

$\boxed{x = -\frac{8}{5}\sin 5t}$ $\quad C=\sqrt{(8/5)^2} = 8/5 \quad \tan\varphi = \dfrac{-8/5}{0}$

$= \varphi = \frac{3\pi}{2}$ as above

and so $\boxed{x = \frac{8}{5}\cos(5t - \frac{3\pi}{2})}$.

(22) $W = mg = 12$ Stretch $s_0 = 0.5$ (6 in.)

$mg = ks_0 \Rightarrow 12 = k \cdot 0.5 = \frac{k}{2} \Rightarrow \boxed{k = 24}$

$c = 3$ given. $m = 0.375$ given: O.D.E. is

$$\boxed{0.375x'' + 3x' + 24x = 0} \qquad \left.\begin{array}{l} x(0) = 1 \\ x'(0) = 0 \end{array}\right\} \text{given.}$$

Char. roots: $0.375r^2 + 3r + 24 = 0$

$$r = \frac{-3 \pm \sqrt{9 - 1.5 \times 24}}{0.75} = -\frac{3}{3/4} \pm \frac{\sqrt{9-36}}{3/4}$$

$$= -4 \pm \frac{\sqrt{27}i}{3/4} = -4 \pm \frac{3\sqrt{3}}{3/4}i = -4 \pm 4\sqrt{3}\,i$$

$$x = c_1 e^{-4t}\cos 4\sqrt{3}t + c_2 e^{-4t}\sin 4\sqrt{3}t$$

$$x' = -4c_1e^{-4t}\cos 4\sqrt{3}t - 4\sqrt{3}c_1 e^{-4t}\sin 4\sqrt{3}t - 4c_2e^{-4t}\sin 4\sqrt{3}t + 4\sqrt{3}c_2e^{-4t}\cos 4\sqrt{3}t$$

$x(0) = c_1 = 1$

$x'(0) = -4c_1 + 4\sqrt{3}c_2 = 0$

$\Rightarrow \begin{array}{l} c_1 = 1 \\ 4\sqrt{3}c_2 = 4 \end{array} \Rightarrow \begin{array}{l} c_1 = 1 \\ c_2 = \frac{1}{\sqrt{3}} \end{array}$

$$\boxed{x(t) = e^{-4t}\cos 4\sqrt{3}t + \frac{1}{\sqrt{3}}e^{-4t}\sin 4\sqrt{3}t}$$

$C = \sqrt{c_1^2 + c_2^2} = \sqrt{4/3} = \frac{2}{\sqrt{3}}$

$\tan\varphi = \frac{c_2}{c_1} = \frac{1}{\sqrt{3}}$

$\varphi = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

or. $\boxed{x(t) = \frac{2}{\sqrt{3}}e^{-4t}\cos(4\sqrt{3}t - \frac{\pi}{3})}$

So frequency is $4\sqrt{3}$, time varying amplitude is

$\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$, phase angle $\varphi = \frac{\pi}{6}$.

3.5) (3) $y'' - y' - 6y = 2\sin 3x$

Char. roots are: $r^2 - r - 6 = 0$ $(r-3)(r+2) = 0$

$r = -2, 3$ $\Rightarrow 2\sin 3x$ not a soln. of $y'' - y' - 6y = 0$

$\Rightarrow$ y_p has form (Rule 1)

$$y_p = A\cos 3x + B\sin 3x.$$

Then $y_p' = -A\,3\sin 3x + 3B\cos 3x$

$y_p'' = -9A\cos 3x - 9B\sin 3x.$

$\Rightarrow y_p'' - y_p' - 6y_p = -9A\cos 3x - 9B\sin 3x - [-3A\sin 3x + 3B\cos 3x] - 6[A\cos 3x + B\sin 3x] = 2\sin 3x$

collect terms on left:

$\Rightarrow$ $(-9A - 3B + 6A)\cos 3x + (-9B + 3A + 6B)\sin 3x = 2\sin 3x$

$\Rightarrow$ $-3A - 3B = 0$ (no cosine terms on right)

$3A - 3B = 2$, + (equating coefficients)

$-6B = 2 \Rightarrow B = -\frac{1}{3}$ $\quad 3A = -3B$ or $A = -B$

$\Rightarrow A = \frac{1}{3}.$

$$\boxed{y_p = \frac{1}{3}\cos 3x - \frac{1}{3}\sin 3x}$$

(9) $y'' + 2y' - 3y = 1 + xe^x$

Characteristic roots are $r^2 + 2r - 3 = 0 \Rightarrow (r-1)(r+3) = 0$, $r = 1, -3$

Basis of solutions for homogeneous problem: e^x, e^{-3x}.

Rule 1 suggests using

$$y_p = A_0 + (B_0 + B_1 x)e^x.$$

However the term $B_0 e^x$ solves the homogeneous eq.

So we have to multiply the 2nd part by x:

$$y_p = A_0 + x(B_0 + B_1 x)e^x = A_0 + B_0 x e^x + B_1 x^2 e^x$$

Now, none of ~~th~~ terms solve $Ly = 0$.

$$y_p' = B_0 e^x + B_0 x e^x + 2B_1 x e^x + B_1 x^2 e^x$$

$$= B_0 e^x + (B_0 + 2B_1) x e^x + B_1 x^2 e^x$$

$$y_p'' = B_0 e^x + (B_0 + 2B_1)e^x + (B_0 + 2B_1) x e^x + 2B_1 x e^x + B_1 x^2 e^x$$

$$= (2B_0 + 2B_1)e^x + (B_0 + 4B_1) x e^x + B_1 x^2 e^x$$

$$y_p'' + 2y_p' - 3y_p = \cancel{A_0}$$

$$(2B_0 + 2B_1)e^x + (B_0 + 4B_1) x e^x + \cancel{B_1 x^2 e^x} +$$

$$2B_0 e^x + 2(B_0 + 2B_1) x e^x + \cancel{2B_1 x^2 e^x}$$

$$- 3A_0 - 3B_0 x e^x - \cancel{3B_1 x^2 e^x}$$

$$= -3A_0 + (4B_0 + 2B_1)e^x + 8B_1 x e^x = 1 + xe^x$$

Equate Coefficients: $-3A_0 = 1$, $4B_0 + 2B_1 = 0$, $8B_1 = 1$ $\Rightarrow$ $A_0 = -\frac{1}{3}$, $B_1 = \frac{1}{8}$, $B_0 = -\frac{1}{2}B_1 = -\frac{1}{16}$

$$\Rightarrow \quad \boxed{y_p = -\frac{1}{3} - \frac{1}{16} x e^x + \frac{1}{8} x^2 e^x}$$

(13) $y'' + 2y' + 5y = e^x \sin x$

Characteristic roots ! $r^2 + 2r + 5 = 0 \Rightarrow r = \frac{-2}{2} \pm \frac{\sqrt{4-20}}{2} = -1 \pm \frac{4i}{2}$

or $r = -1 \pm 2i$

Rule 1 suggests $y_p = A e^x \cos x + B e^x \sin x$. Neither term satisfies the homogeneous eq. so we ~~use~~ use this for y_p

$$y_p' = A e^x \cos x - A e^x \sin x + B e^x \sin x + B e^x \cos x$$

$$y_p'' = \cancel{A e^x \cos x} - A e^x \sin x - A e^x \sin x - \cancel{A e^x \cos x}$$
$$+ \cancel{B e^x \sin x} + B e^x \cos x + B e^x \cos x - \cancel{B e^x \sin x}$$

$$= 2B e^x \cos x - 2A e^x \sin x$$

$$y_p'' + 2y_p' + 5y_p = 2B e^x \cos x - 2A e^x \sin x + 2A e^x \cos x - 2A e^x \sin x + 2B e^x \sin x$$
$$+ 2B e^x \cos x + 5A e^x \cos x + 5B e^x \sin x$$

$$= (4B + 7A) e^x \cos x + (-4A + 7B) e^x \sin x = e^x \sin x$$

Equating coeff's: $\begin{aligned} 4B + 7A &= 0 \\ -4A + 7B &= 1 \end{aligned} \Rightarrow A = -\frac{4}{7} B$

$$-4\left(-\frac{4}{7} B\right) + 7B = 1 \Rightarrow \frac{16}{7} B + \frac{49}{7} B = 1$$

$\cancel{B = -\frac{58}{7}}$ $\cancel{A = -\frac{4}{7}\cdot\left(-\frac{58}{7}\right) = \frac{232}{49}}$

$\frac{65}{7} B = 1$

$\cancel{y_p = -\frac{232}{49} e^x \cos x - \frac{58}{7} e^x \sin x}$

$B = \frac{7}{65}$ $\quad A = -\frac{4}{7} \cdot \frac{7}{65} = -\frac{4}{65}$

$$\boxed{y_p = \frac{-4}{65} e^x \cos x + \frac{7}{65} e^x \sin x}$$

(20) $y^{(3)} - y = e^x + 7$

Char. roots $r^3 - 1 = 0 \Rightarrow (r-1)(r^2+r+1)$

$r = 1, \quad r = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

Rule 1 suggests $y_p = A_0 + B_0 e^x$. However $B_0 e^x$ satisfies the homogeneous equation, so we multiply by x (the smallest power of s of x such that $B_0 x^s e^x$ not a soln. of $Ly = 0$). So

$y_p = A_0 + B x e^x$

$y_p' = B e^x + B x e^x$

$y_p'' = B e^x + B e^x + B x e^x = 2B e^x + B x e^x$

$y_p''' = 2B e^x + B e^x + B x e^x = 3 B e^x + B x e^x$

$y_p''' - y_p = 3B e^x + \cancel{B x e^x} - (A + \cancel{B x e^x})$

$= -A + 3B e^x = 7 + e^x$

$\Rightarrow \begin{matrix} -A = 7 \\ 3B = 1 \end{matrix} \Rightarrow \begin{matrix} A = -\frac{1}{7} \\ B = \frac{1}{3} \end{matrix}$

$$\boxed{y_p = -\frac{1}{7} + \frac{1}{3} x e^x}$$

(27) $y^{(4)} + 5y'' + 4y = \sin x + \cos 2x$

Characteristic roots: $r^4 + 5r^2 + 4 = 0$

$\Rightarrow (r^2+4)(r^2+1) = 0$

$\Rightarrow r^2 = -4 \quad \Rightarrow \quad r = \pm 2i$

$r^2 = -1 \qquad\qquad r = \pm i.$

Basis of solutions for homogeneous problem:

$\cos x, \sin x, \cos 2x, \sin 2x.$

Rule 1 suggests $y_p = A\cos x + B\sin x + C\cos 2x + D\sin 2x$

however each term satisfies homogeneous problem; if we multiply these by x, they would not. Therefore:

$$\boxed{y_p = Ax\cos x + Bx\sin x + Cx\cos 2x + Dx\sin 2x}$$

or $$\boxed{y_p = x(A\cos x + B\sin x) + x(C\cos 2x + D\sin 2x)}$$

Sec. 3.6 (2) $x'' + 4x = 5\sin 3t, \quad x(0) = 0, \quad x'(0) = 0$

$x_p = A\cos 3t + B\sin 3t$

$x_p' = -3A\sin 3t + 3B\cos 3t$

$x_p'' = -9A\cos 3t - 9B\sin 3t$

$x_p'' + 4x_p = -9A\cos 3t - 9B\sin 3t + 4A\cos 3t + 4B\sin 3t$

$= -5A\cos 3t - 5B\sin 3t = 5\sin 3t$

$\Rightarrow -5A = 0, \quad -5B = 5$

$\Rightarrow A = 0, \quad B = -1$

$\Rightarrow \boxed{x_p = -\sin 3t}$

Characteristic roots: $r^2 + 4 = 0 \Rightarrow r = \pm 2i.$

Solution $\cos 2t$, $\sin 2t$ so

$x = c_1 \cos 2t + c_2 \sin 2t - \sin 3t$

$x' = -2c_1 \sin 2t + 2c_2 \cos 2t - 3\cos 3t$

$x(0) = c_1 = 0 \qquad \Rightarrow \qquad c_1 = 0$

$x'(0) = 2c_2 - 3 = 0 \qquad 2c_2 = 3 \qquad c_2 = \frac{3}{2}$

$\boxed{x(t) = \frac{3}{2}\sin 2t - \sin 3t}$

(8) $x'' + 3x' + 5x = -4\cos 5t.$

x_{sp} is just the particular solution found by method of undetermined coefficients.

$x_{sp} = A\cos 5t + B\sin 5t \qquad$ (neither term solves homog. problem)

$x_{sp}' = -5A\sin 5t + 5B\cos 5t$

$x_{sp}'' = -25A\cos 5t - 25B\sin 5t$

$x_{sp}'' + 3x_{sp}' + 5x_{sp} = -25A\cos 5t - 25B\sin 5t + -15A\sin 5t + 15B\cos 5t$

$+ 5A\cos 5t + 5B\sin 5t$

$= (-20A + 15B)\cos 5t + (-15A - 20B)\sin 5t$

$$-20A + 15B = -4 \qquad\qquad\qquad -20A + 15B = -4$$
$$-15A - 20B = 0 \quad \Rightarrow \quad -3A - 4B = 0 \Rightarrow A = -\tfrac{4}{3}B$$

$$-20\left(-\tfrac{4}{3}B\right) + 15B = -4$$

$$\left(\frac{80}{3} + \frac{45}{3}\right)B = -4, \quad \frac{125}{3}B = -4 \quad B = -\frac{12}{125}$$

$$A = -\frac{4}{3}\left(-\frac{12}{125}\right) = \frac{16}{125}$$

$$x_{sp} = \frac{16}{125}\cos 5t - \frac{12}{125}\sin 5t$$

$$C = \sqrt{A^2+B^2} = \sqrt{\frac{16^2+12^2}{125^2}} = \sqrt{\frac{400}{125^2}} = \frac{20}{125} = \frac{4}{25}$$

$$\tan\varphi = \frac{B}{A} = \frac{-12/125}{16/125} = -\frac{3}{4} \Rightarrow \varphi = \tan^{-1}\left(-\tfrac{3}{4}\right) + 2\pi = 5.64.$$

$$x = \frac{4}{25}\cos(5t - 5.64)$$

(12) $x'' + 6x' + 13x = 10\sin 5t \qquad x(0) = 0, \; x'(0) = 0$

$$x_{sp} = x_p = A\cos 5t + B\sin 5t$$
$$x_{sp}' = -5A\sin 5t + 5B\cos 5t$$
$$x_{sp}'' = -25A\cos 5t - 25B\sin 5t$$

$$x_{sp}'' + 6x_{sp}' + 13x_p = -25A\cos 5t - 25B\sin 5t - 30A\sin 5t + 30B\cos 5t$$
$$+ 13A\cos 5t + 13B\sin 5t$$
$$= (-12A + 30B)\cos 5t + (-30A - 12B)\sin 5t = 10\sin 5t$$

$$\Rightarrow \quad -12A + 30B = 0 \qquad -2A + 5B = 0 \qquad \Rightarrow \qquad A = \frac{5}{2}B$$

$$-30A - 12B = 10 \qquad -15A - 6B = 5 \qquad -15 \cdot \frac{5}{2}B - 6B = 5$$

$$\Rightarrow \quad -\frac{87}{2}B = 5 \quad \Rightarrow \quad B = -\frac{10}{87} \qquad A = -\frac{5}{2}\,\frac{10}{87} = -\frac{25}{87}$$

$$x_{sp} = -\frac{25}{87}\cos 5t - \frac{10}{87}\sin 5t$$

$$C = \sqrt{A^2 + B^2} = \sqrt{\frac{25^2 + 10^2}{87^2}} = \frac{\sqrt{725}}{87} = \frac{5\sqrt{29}}{87}$$

$$\tan\varphi = \frac{B}{A} = \frac{-10/87}{-25/87} = \frac{2}{5} \qquad \varphi = \tan^{-1}(2/5) + \pi = 3.52$$

$$\boxed{x_{sp} = \frac{5\sqrt{29}}{87}\cos(5t - 3.52)} \qquad \left(\text{note: } \frac{5\sqrt{29}}{87} = \frac{5}{3\sqrt{29}}\right)$$

Characteristic roots

$$r^2 + 6r + 13 = 0 \quad \Rightarrow \quad r = -\frac{6}{2} \pm \frac{\sqrt{36 - 52}}{2} = -3 \pm 2i$$

$$\Rightarrow \quad x = c_1 e^{-3t}\cos 2t + c_2 e^{-3t}\sin 2t + \cancel{\frac{5\sqrt{29}}{87}\cos(5t - 3.52)}$$
$$+ -\frac{25}{87}\cos 5t - \frac{10}{87}\sin 5t$$

$$x' = -3c_1 e^{-3t}\cos 2t - 2c_1 e^{-3t}\sin 2t - 3c_2 e^{-3t}\sin 2t + 2c_2 e^{-3t}\cos 2t$$
$$+ \frac{75}{87}\sin 5t - \frac{50}{87}\cos 5t$$

$$x(0) = c_1 - \frac{25}{87} = 0 \quad \Rightarrow \quad c_1 = \frac{25}{87}$$

$$x'(0) = -3c_1 + 2c_2 - \frac{50}{87} = 0 \qquad -3 \cdot \frac{25}{87} + 2c_2 = \frac{50}{87} \quad \Rightarrow \quad 2c_2 = \frac{125}{87}$$

$$C_2 = +\frac{125}{2\cdot 87} = \frac{+125}{174}$$

$$X_{tr} = c_1 e^{-3t}\cos 2t + c_2 e^{-3t}\sin 2t = C e^{-3t}\cos(2t-\varphi)$$

where $C = \sqrt{c_1^2 + c_2^2} = \sqrt{\frac{25^2}{87^2} + \frac{125^2}{2^2\cdot 87^2}} = \sqrt{\frac{4\cdot 25^2 + 125^2}{4\cdot 87^2}} =$

$$= \frac{\sqrt{18125}}{2\cdot 87} = \frac{5\sqrt{725}}{2\cdot 87} = \frac{25\sqrt{29}}{2\cdot 87}\,\frac{\sqrt{29}}{\sqrt{29}} = \frac{25\cdot \cancel{29}}{\cancel{2}\cdot 3\cdot \cancel{29}\cdot\sqrt{29}}$$

$$= \boxed{\frac{25}{6\sqrt{29}}} \qquad \tan\varphi = \frac{c_2}{c_1} = \frac{125/174}{25/87} = +\frac{5}{2}$$

$$\varphi = \tan^{-1}\frac{5}{2} = 1.19$$

$$\boxed{X_{tr} = \frac{25}{6\sqrt{29}} e^{-3t}\cos(2t - 1.19)}$$

(19) $W = mg = k s_0$ $\quad s_0 =$ stretch $\Rightarrow 100\text{lb} = \frac{k}{12}$ (in ft.)

~~$m = 3.125$ so~~

$$\Rightarrow \quad mg = k/12 \quad \text{so} \quad \frac{k}{m} = 12\cdot g \qquad \text{But } \frac{k}{m} = \omega_0^2$$

$\Rightarrow \omega_0 = \sqrt{12\cdot 32} = \boxed{\sqrt{384}}$. This is the resonance frequency