Week 3 Examples

Example 1: Solve: (1) $y'' = -9.8$, (2) $y'' = -0.04y' - 9.8$, both with $y(0) = 0$, $y'(0) = 49$.
Answers: (1) $y = -9.8t^2/2 + 49t$; (2) $y = 7350 - 245t - 7350e^{-t/25}$.

Example 2: Let $w = v\sqrt{\rho/g}$ and $p = \frac{1}{\sqrt{g\rho}}$ to replace Newton's quadratic drag model $v' = -g - \rho v|v|$ by $pw' = -1 - w|w|$. Explain rise model $pw' = -1 - w^2$ and fall model $pw' = -1 + w^2$.

Example 3: Solve $pw' = -w^2 + 1$ and $pw' = w^2 + 1$ as separable equations. See the previous example. **Answers**: $w(t) = \tanh(c_1 + t/p)$ and $w(t) = \tan(c_2 + t/p)$

Example 4: Verify rise time 4.6 and fall time 4.8 for Newton's quadratic drag model $v' = -9.8 - 0.0011v|v|$, $v(0) = 49$. Use textbook formulas or the previous two examples.

Example 5: Find the point $r = r^*$ of zero acceleration in the Jules Verne equation $r'' = -\frac{Gm_1}{(R_1+r)^2} + \frac{Gm_2}{(R_3-r)^2}$. The answer has symbols. Then calculate $r^* \approx 339,620,820$ meters for the earth-moon problem. Reference:
http://www.math.utah.edu/~gustafso/s2016/2280/julesVerneDE2008.pdf