Week 1 Examples

Three Examples. Solve differential equations without a book. Three basic examples used throughout a course in differential equations, which require only a calculus background.

Growth-Decay: $\dfrac{dA(t)}{dt} = k\,A(t),\ A(0) = A_0.$
The unique solution is $A(t) = A_0\,e^{kt}$. Radioactive decay. Jeweler's bench light experiment. Malthusian population dynamics. RC and LR circuits. Drug elimination. First-order chemical reactions, law of mass-action. Compound continuous bank interest.

Newton Cooling: $\dfrac{du(t)}{dt} = -h(u(t) - u_1),\ u(0) = u_0.$
The solution is $u(t) = u_1 + (u_0 - u_1)\,e^{-ht}$. Hot chocolate at initial temperature u_0 with room thermometer reading u_1. Symbol $u(t) =$ time-varying hot chocolate dial thermometer temperature.

Verhulst Dynamics: $\dfrac{dP(t)}{dt} = (a - b\,P(t))\,P(t),\ P(0) = P_0.$
The solution is $P(t) = \dfrac{aP_0}{bP_0 + (a - bP_0)e^{-at}}$. Fish population $P(t)$ in **Cecret Lake** at **Alta**. Carrying capacity. Stocking and re-stocking. Harvesting.

Example 1: Exercise 1.2-2: Solve $dy/dx = (x-2)^2,\ y(2) = 1$.
Method of quadrature. Answer Check details. Non-reversible steps, false proof for $0 = 1$.
http://www.math.utah.edu/ gustafso/s2016/2280/FTC-Method-of-Quadrature.pdf

Example 2: Exercise 1.2-4: $dy/dx = 1/x^2, y(1) = 5$.
Power rule in Newton calculus. Answer check shortcuts.

Example 3: Exercise 1.2-10: $dy/dx = x\,e^{-x}, y(0) = 1$.
Integral tables and integration by parts. Jennifer Lahti's solution:
http://www.math.utah.edu/~gustafso/s2016/2280/2250Week1exercises-JenniferLahti-1.2-5+8+10.pdf

Example 4: Exercise 1.3-8: $dy/dx = x^2 - y$
Thread edge-to-edge solutions through the direction field at each blue dot. JPEG image source:
http://www.math.utah.edu/~gustafso/s2016/2280/exercise1.3-8-EdwardsPenney.jpg

Example 5: Exercise 1.3-14: $dy/dx = y^{1/3}, y(0) = 0$
Explain application of the Peano and Picard theorems.
Computer numerical methods fail on this example. Why?

Example 6: Exercise 1.4-6: Solve $y' = 3\sqrt{xy}$
Three answers. Book reports only one answer.

Example 7: Exercise 1.4-10: Solve $(1 + x^2)y' = (1 + y)^2$
Two answers. Book reports only one answer.

Example 8: Exercise 1.4-18: Solve $x^2y' = 1 - x^2 + y^2 - x^2y^2$ [See Example 11 *infra*]

Example 9: Exercise 1.4-22: Solve $y' = 4x^3y - y, y(1) = -3$

Example 10: Show that $y' = x + y$ is not separable.
TEST I. f_x/f depends on y implies $y' = f(x, y)$ not separable.

Example 11: Find a factorization $f(x, y) = F(x)G(y)$ given
(1) $f(x, y) = 2xy + 4y + 3x + 6$
(2) $f(x, y) = (1 - x^2 + y^2 - x^2y^2)/x^2$
Answers: (1) $F = x + 2, G = 2y + 3$; (2) $F = (1 - x^2)/x^2, G = 1 + y^2$. Main idea: Choose $y = 0$ in $F(x) = f(x, y)/G(y)$ to find $F(x) = (3x + 6)/G(0)$ in equation (1). How to find G? **Warning**: Divide by zero is not allowed. Choose $y = 0$, $y = 1$, etc, until no divide by zero error.

Example 12: Midterm 1 examples: $y' = x + y, y' = x + y^2, y' = x^2 + y^2$