The Illustrated Tales of the Vertex Adjacency Matrix

by Deni Weeks

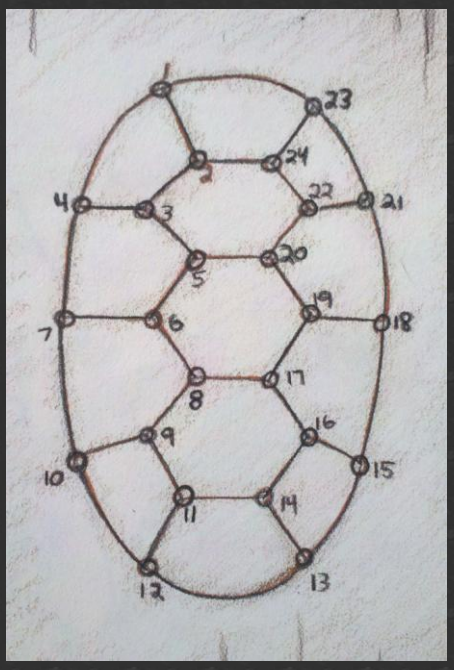
Some stories begin with a chance meeting at a crossroads or a unbeaten path branching off from a fork in the road. And roads less traveled are those that inspire the tale. And as readers, we are often advised to look more kindly upon the journey than the destination. I entreat any traveler here to do the same.

The Good Ship Testudinidae

Every beetle knows that the tortoise is the luxury liner of the animal kingdom. If you have no particular place to go or in no particular hurry, a tortoise is the way to travel. And every cruise ship should be explored for all its comforts and amenities. But the tortoise isn't well known for its shuffle boards and dining halls. And the view from any particular point on the tortoise is much like the rest. So to entertain himself, our beetle counts his paces and applies his mathematical acumen to make his journey more interesting. He counts the intersections on his host's carapace. There are 24 and he calls them vertices.

He doesn't count the little ones on the edge, because he might fall off. And for each vertex he creates a new row in a matrix. And each entry in the row corresponds to all the other vertices. If

he can travel from his current vertex to any other by a length of one, he gives the entry a 1 and the rest only get zeros. He continues on until all the vertices are accounted for and he has a matrix 24 by 24. He calls this matrix T. Because T is for Tortoise.



$$T := \begin{bmatrix}
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1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
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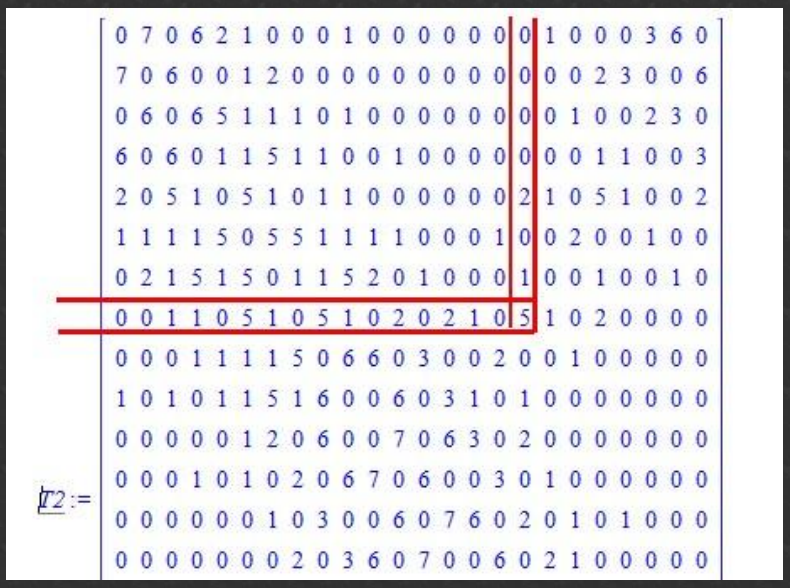
\end{bmatrix}$$

But this alone is not useful enough, says our beetle. He wishes to go from vertex 8 to vertex 17 by 3 lengths, how then should he proceed? And what other vertices may he travel in lengths of three? The tortoise knows his linear algebra and he knows the surest way to calculate is by powers of T. And he knows that T to the power of 3 will reveal the number of paths length three that exist between any two vertices.

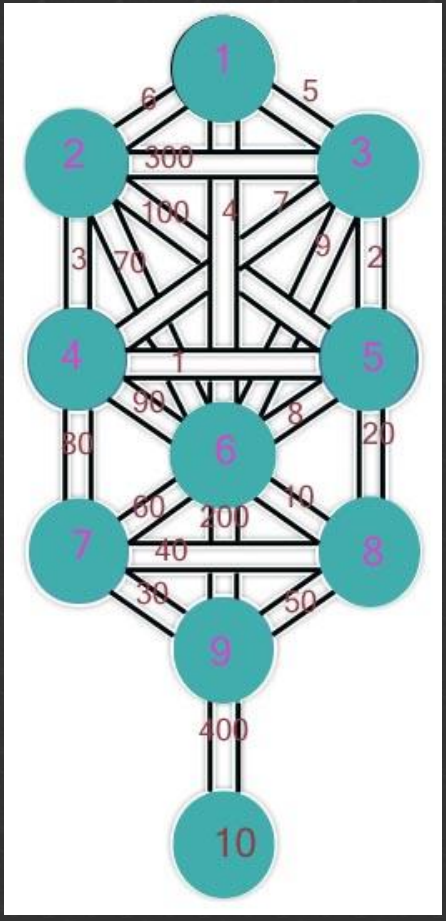


And so our beetle follows along row 8 until he finds the entry in the 17th column. There he discovers that there are five paths with length three that connect vertex 3 and 23. And he finds them.

And he continues on his calculations and his walks of lengths, his host carries him on into the landscape. Of course as smart as our beetle friend is, he has yet to realize he has traveled in magnitudes greater than would have taken him if he had set out to travel by himself.

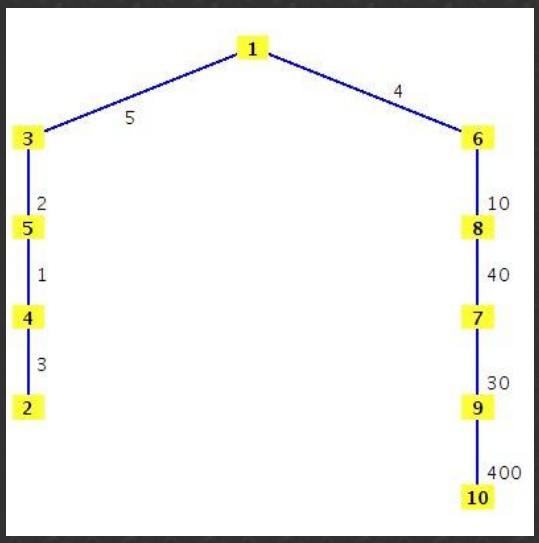
Sara and the Spanning Tree Theology

Little Sara saw her first sephirah emblazoned in bright colors with mystic drawings in a book of Kabbalah mysteries in Papa Sal's library. As she read, she discovered that it was a divine path. But a path should have a starting trailhead and markers that led you correctly down forked paths and the sephirah had none. But as little Sara studied, the sephirah revealed itself as a graph of vertices, undirected. She saw the edges weighted by Hebrew letters that she knew were also numbers. And she set out to label every vertex on the sephirah and note the weights of every edge. And she made an adjacency matrix accordingly.



$$A := \begin{bmatrix} 0 & 6 & 5 & 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 6 & 0 & 300 & 3 & 100 & 70 & 0 & 0 & 0 & 0 \\ 5 & 300 & 0 & 7 & 2 & 9 & 0 & 0 & 0 & 0 \\ 0 & 3 & 7 & 0 & 1 & 90 & 80 & 0 & 0 & 0 \\ 0 & 100 & 2 & 1 & 0 & 8 & 0 & 20 & 0 & 0 \\ 4 & 70 & 9 & 90 & 8 & 0 & 60 & 10 & 200 & 0 \\ 0 & 0 & 0 & 80 & 0 & 60 & 0 & 40 & 30 & 0 \\ 0 & 0 & 0 & 0 & 20 & 10 & 40 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 0 & 200 & 30 & 50 & 0 & 400 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 400 & 0 \end{bmatrix}$$

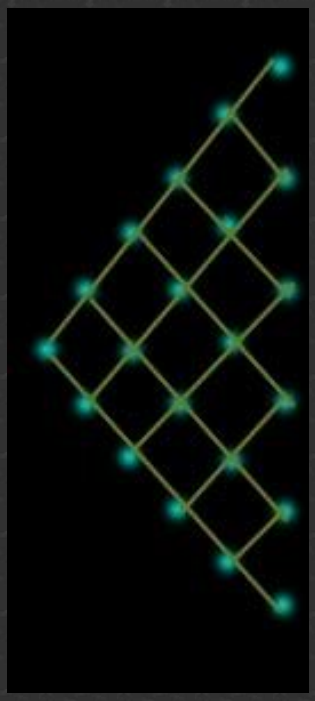
She could sit for hours and draw each of the differently oriented paths that would connect each vertex. Each of them a spanning tree. But only one of them would cost the least based on the weights of all the edges. And she knew at once that Mr. Prim would make everything proper even though he was indeed a greedy algorithm. Prim would find the tree that would cost the least, the path that should be the quickest way to the divine.



And there it was, the path that was not where she imagined. She smiled and tucked away her graphs and matrices into the page with the sephirah for Papa Sal to find.

The Equipoise Eel

The Eel lives a quiet life, but occasionally the fates conspire to make his life much more interesting than he would like it. One particular eel found himself at the height of excitement, at the highest point on the great wave of Kanagawa. Luckily an eel can think quickly on their non-feet. But unluckily, there was nothing that could be done. He was going to be washed in whatever random direction would have him. But luckily again, the eel was a fatalist. There was nothing left but to imagine what his fate could be. He imagined discrete points in his journey and at that point an ever-increasing number of vertices adjacent to his previous point. For a moment the eel allowed himself to be jealous of feet-having kinds who could simulate this same journey with a random, drunken walk through a poorly lit city. But our eel, he digresses. And so each point he represents with a column in a matrix. And each entry in the column represents the probability of traveling to an adjacent vertex from the previous point. And because our eel was equipoise, the probability was distributed uniformly over all possible vertices.



$$E := \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} & \frac{1}{4} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

The eel is quick to point out that this is indeed a Markov matrix since each column adds to 1, which conveniently in this case is the total probability for each successive step in our random journey. Our eel is also very wise to observe that even given the same original starting point, repeated journeys would yield vastly different results. There wasn't a way to guess how long it would be before gravity snapped the wave and his footless body back into the ocean. And luckily it would, because such a process could be imagined on an infinite scale, with an infinite matrix. Of course, he would have no time to ponder such things because he never made it back to the ocean when the wave very randomly assigned his fate to the bottom of a fishing boat where lucky fisherman were delighted to have their dinners handed to them.