Sitaram Gautam

MATH 2270 Linear Algebra
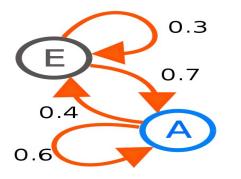
Semester Project

Professor Grant Gustafson

May 1, 2012

Markov Chain and its application in Voting Behavior

What is Markov Chain?

A Markov chain, named after Andrey Markov, is a mathematical system that undergoes transitions from one state to another, between a finite or countable number of possible states. It is a random process characterized as memoryless: the next state depends only on the current state and not on the sequence of events that preceded it. The picture alongside shows a simple two-state Markov Chain.
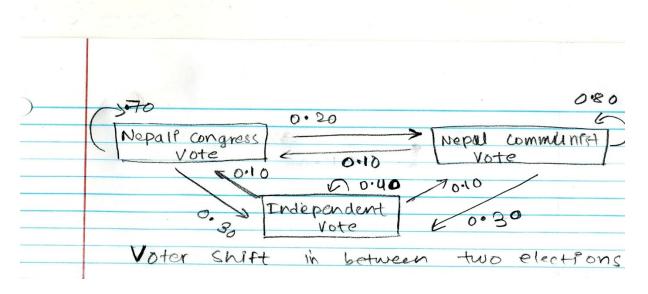
What is Markov Matrix?

If the probability of moving from i to j in one time step is $Pr(j|i) = P_{i,j}$, the Markov matrix P is given by using $P_{i,j}$ as the ith row and j^{th} column element, e.g.,

$$P = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,j} & \cdots \\ p_{2,1} & p_{2,2} & \cdots & p_{2,j} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \ddots \\ p_{i,1} & p_{i,2} & \cdots & p_{i,j} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \ddots \end{pmatrix}.$$

Markov matrix is also called stochastic matrix. A simple example of a Markov matrix is given the picture below, the matrix P.

Markov chains can be used to model situations in many fields, including biology, chemistry, economics and finance. As an example of Markov chain application, consider Voting behavior. A population of voters is distributed between the Nepali Congress (NC), Nepal Communist Party (NCP), and Independent (I) parties. Each election, the voting population p = [NC, NCP, I] obeys the redistribution shown in the figure below.

For example, in an upcoming election, of those that voted Nepal Communist Party in the previous election, 80% will remain Nepal Communist, 10% will vote Nepali Congress, and the remaining 10% will vote Independent.

The transition matrix describing this tendency of voter shift is given below.

Party

$$T = \begin{array}{c} NC \\ NCP \\ I \end{array} \begin{array}{ccc} NC & NCP & I \end{array} \left[\begin{array}{ccc} 0.70 & 0.10 & 0.30 \\ 0.20 & 0.80 & 0.30 \\ 0.10 & 0.10 & 0.40 \end{array} \right]$$

$$D_2 = T D_1 \left[\begin{array}{ccc} 0.70 & 0.10 & 0.30 \\ 0.20 & 0.80 & 0.30 \\ 0.10 & 0.10 & 0.40 \end{array} \right] \left[\begin{array}{c} 0.48 \\ 0.51 \\ 0.1 \end{array} \right]$$

$$= \left[\begin{array}{c} 0.390 \\ 0.507 \\ 0.103 \end{array} \right]$$
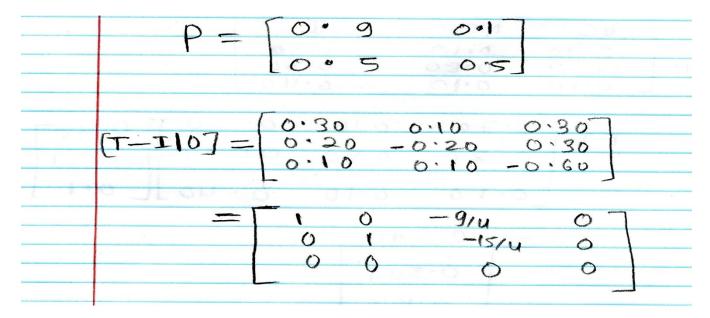
The column labels indicate that the initial party of a portion of the voting population, while the row labels indicate the final party. For example in column NCP, row NC we see that 10% of those who previously voted Nepal Communist Party will vote Independent.

In the 2004 constitutional election, the voters where distributed according to the distribution vector D = <0.48, 0.51, 0.01>. If the transition matrix above dictates the changes between two primary elections, we can expect the outcome of future election is given as matrix D2 in above picture.

More explicitly, 70% of the original Nepali Congress (48%) remains Nepali Congress, 10% of the 51% Communist will vote Nepali Congress, and 30% of the 1% Independent population also votes Congress

.70(.48) + .10(.51) + .30(.01) = .336 + .051 + .003 = .390

The same goes for the shift of votes to the Communist and Independent parties.

If this voter behavior is valid for a long time, we can predict the outcome of a given election by applying the transition matrix the appropriate number of times to the initial distribution vector. Particularly, it can be shown that since the transition matrix here is obviously

regular, a steady-state solution is expected. Using the row-reduction method mentioned above to solve the system $[T - I\,|0]$, we can solve for the steady state vector.

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix}$$

$$[T - I\,|0] = \begin{bmatrix} 0.30 & 0.10 & 0.30 \\ 0.20 & -0.20 & 0.30 \\ 0.10 & 0.10 & -0.60 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -9/4 & 0 \\ 0 & 1 & -15/4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The eigenvector of this system corresponding to eigenvalue $A = 1$ therefore, has components $X1 = 9/4$, $X2 = 15/4$, $X3 =$ free. In order to transform this into a valid population vector (with the sum of the entries equal to 1), the entries are summed and then each is divided by the result. The vector s therefore demonstrates that the voting population will eventually settle into a state in which 54% of the votes will be cast for the Communist candidate.

S = < 9/28, 15/28, 1/7> ~ <0.32, 0.54, 0.14>.