Name ______________________________ Class Time ______

Math 2270 Extra Credit Problems
Chapter 1
December 2011

These problems were created for Bretscher's textbook, but apply for Strang's book, except for the division by chapter. To find the background for a problem, consult Bretscher's textbook, which can be checked out from the math library or the LCB Math Center.

Due date: See the internet due dates. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem XC1.1-26. (Three possibilities)

Determine which values of k correspond to (a) a unique solution, (b) no solution or (c) infinitely many solutions.

$$\left|\begin{array}{rcrcrcr} x & + & 2y & + & kz & = & 0 \\ 2x & + & 4y & + & (k+1)z & = & 2 \\ 3x & + & 6y & + & (2k+1)z & = & 2 \end{array}\right|$$

Problem XC1.1-30. (Polynomial interpolation)

Find the polynomial $f(x) = a + bx + cx^2$ which passes through the points $(1, 9)$, $(2, 24)$, $(3, 47)$.

Problem XC1.1-32. (Polynomial interpolation)

Find all polynomials $f(x) = a + bx + cx^2$ which pass through the points $(1, 10)$, $(2, 28)$ and $f'(3) = 33$.

Problem XC1.2-22. (RREF)

Report five types of 3×4 matrices in RREF form.

Problem XC1.2-28. (Combo rule)

Consider the following systems.

$$(1) \qquad \left|\begin{array}{ccccccc} a_{11}x_1 & + & \cdots & + & a_{1n}x_n & = & b_1 \\ & & \vdots & & & & \\ a_{m1}x_1 & + & \cdots & + & a_{mn}x_n & = & b_m \end{array}\right|$$

$$(2) \qquad \left|\begin{array}{ccccccc} a_{11}x_1 & + & \cdots & + & a_{1n}x_n & = & b_1 \\ & & \vdots & & & & \\ c_{k1}x_1 & + & \cdots & + & c_{kn}x_n & = & d_k \\ & & \vdots & & & & \\ a_{m1}x_1 & + & \cdots & + & a_{mn}x_n & = & b_m \end{array}\right|$$

We assume the systems identical except for equation k, which is obtained in the second system by applying a combination rule `combo(r,k,c)` to the first system. We assume $r \neq k$ and then the coefficients in the second system are given by

$$c_{kj} = a_{kj} + ca_{rj}, \quad j = 1, \ldots, n, \quad d_k = b_k + cb_r.$$

(a) Prove that every solution of system (1) is a solution of system (2).

(b) Prove that every solution of system (2) is a solution of system (1).

Problem XC1.2-30. (Polynomial interpolation)

Find the polynomial $f(x) = a + bx + cx^2 + dx^3$ which satisfies $f(1) = 8$, $f(2) = 24$, $f(3) = 24$, $f(4) = 110$.

Problem XC1.3-26. **(Matrix algebra)**

Find a 3×3 matrix A which satisfies the following relations.

$$A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

End of extra credit problems chapter 1.