Week 2 Examples

Example 1: Classify as separable (S), quadrature (Q), linear (L) or none (N). (1) $y' = 3(xy)^{1/3}$, (2) $y' = xy^2 + 1$, (3) $y' = x\sin(y)$, (4) $y' = y\sin(x)$, (5) $y' = e^{\ln|x|}$, (6) $y' + xy = x^2 y$
Answers: (1) S; (2) N; (3) S; (4) S,L; (5) Q,S,L; (6) L.

Example 2: Check **explicit** answer $y = (x^{3/2} + c)^2$ for $y' = 3\sqrt{x}\sqrt{y}$ on domain $x \ge 0$, $y \ge 0$.

Example 3: Check **implicit** answer $\csc(y)\cot(y) = -x^2/2 + c$ for $y' = x\sin(y)$.

Example 4: Let $f(x, y) = 1 - x^2 + y^2 - x^2y^2$. In relation $f(x, y) = F(x)G(y)$, equations $f(x, 0) = F(x)G(0), f(0, y) = F(0)G(y)$ can determine F, G . Explain. Then find one pair F, G.

Example 5: Solve using the constant equation shortcut or the quadrature shortcut.
(1) $y' + 2y = 6$, (2) $2y' + 5y = 3$, (3) $2y' = 3$, (4) $3y' = 5y + \pi$.

Example 6: Solve using the integrating factor shortcut for homogeneous equations.
(1) $y' + 8xy = 0$, (2) $2y' + \sin(x)y = 0$, (3) $xy' + \ln|x|y = 0$.

Example 7: Solve a non-separable equation using the integrating factor method.
(1) $xy' + 2y = x^2$, (2) $xy' + 2y = x$, (3) $xy' + 2y\ln|x| = \ln|x|\, e^{(\ln|x|)^2}$.
Answers: (1) $y = x^2/4 + c/x^2$, (2) $y = x/3 + c/x^2$, (3) $y = \frac{1}{4}\, e^{(\ln|x|)^2} + c/e^{(\ln|x|)^2}$.

Example 8: Solve the brine tank model $\frac{dx}{dt} = 1/4 - x/16$, $x(0) = 20$.

Example 9: Solve the brine tank cascade $x' = -x/2$, $y' = x/2 - y/4$, $z' = y/4 - z/6$ with $x(0) = 1, y(0) = -2, z(0) = 1.5$. **Answer**: $x = e^{-t/2}$, $y = -2e^{-t/2}$, $z = 1.5e^{-t/2}$

Example 10: Find all equilibrium solutions for $(x^2 + 1)y' = x + 1 - xy^2 - y^2$

Example 11: Solve $y' = (1 - y)y$ by the substitution $u = y/(1 - y)$.

Example 12: Solve $y' = (1 - y)y$ by partial fraction methods. Check the answer from $P' = (a - bP)P$ and the Verhulst formula $P = \dfrac{aP_0}{bP_0 + (a - bP_0)e^{-at}}$.

Example 13: Assume US population data $5.308, 23.192, 76.212$ million for years $100, 1850, 1900$, respectively. Find a, b in the Verhulst model $P' = (a - bP)P$. Answer: $a = 0.3155090164$, $b = 0.00167716$.

Example 14: Solve $y' = 7y(y - 13)$, $y(0) = 17$. See 2.1-8.

Example 15: Draw a phase line diagram for $y' = y(1 - y)^2(y + 1)$. See Section 2.2.

Example 16: Draw a phase diagram for $y' = y^2(y^2 - 4)$. See 2.2-17.

Example 17: Justify why the direction field along a line $x = x_0$ is the same as the direction field along $x = 0$, for any autonomous equation $y' = F(y)$.