Linear Algebra 2270-2

Due in Week 7

The seventh week finishes the work from chapter 3. Here's the list of problems, followed by problem notes and a few answers.

Section 3.5. Exercises 2, 5, 9, 10, 11, 15, 18, 23, 26, 31, 45

Section 3.6. Exercises 1, 2, 5, 7, 11, 24

Problem Notes

Issues for Strang's problems will be communicated here. For help, please send email, call 581-6879, or visit JWB 113.

Some Answers

3.5. Exercises 2, 11, 15, 18 have textbook answers.

3.5-5. (a) Reduce to upper triangular $\begin{pmatrix} 1 & 2 & 3 \\ 0 & -5 & -7 \\ 0 & 0 & -18/5 \end{pmatrix}$. Invertible implies independent columns.

(b) Reduce to upper triangular $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 7 & -7 \\ 0 & 0 & 0 \end{pmatrix}$. Then $A\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, columns add to zero.

3.5-9. a) The four vectors in $\mathcal{R}^3$ are the columns of a 3 by 4 matrix A. There is a nonzero solution to $Ax = 0$ because there is at least one free variable

(b) Two vectors are dependent if $[v_1|v_2]$ has rank 0 or 1. (OK to say "they are on the same line" or "one is a multiple of the other" but not v_2 is a multiple of 1 – since v_1 might be 0.)

(c) A nontrivial combination of v_1 and 0 gives 0: $0\vec{v}_1 + 3\vec{0} = \vec{0}$.

3.5-10. The plane is the nullspace of 1 by 4 matrix $A = \begin{bmatrix} 1 & 2 & -3 & -1 \end{bmatrix}$. Three free variables give three solutions $(x, y, z, t) = (2, -1, 0, 0)$, $(3, 0, 1, 0)$ and $(1, 0, 0, 1)$. Combinations of those special solutions give more solutions (all solutions).

3.5-23. Columns 1 and 2 are bases for the (different) column spaces of A and U; rows 1 and 2 are bases for the (equal) row spaces of A and U; $(1, -1, 1)$ is a basis for the (equal) nullspaces

3.5-26.

(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) Add $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$

These are simple bases (among many others) for (a) diagonal matrices (b) symmetric matrices (c) skew-symmetric matrices. The dimensions are $3, 6, 3$.

3.5-31. (a) $y(x) =$ constant C (b) $y(x) = 3x$ (c) $y(x) = 3x + C = y_p + y_n$ solves $dy/dx = 3$.

3.5-45. If the left side of $\dim(V)+\dim(W) = \dim(V\cap W)+\dim(V+W)$ is greater than n, then $\dim(V+W) \leq n$ implies $\dim(V \cap W)$ must be greater than zero. So $V \cap W$ contains nonzero vectors.

3.6. Exercises 1, 11, 24 have textbook answers.

3.6-2.

A: Row space basis = row 1 =$(1, 2, 4)$; nullspace $(-2, 1, 0)$ and $(-4, 0, 1)$; column space basis = column 1 = $(1, 2)$; left nullspace $(-2, 1)$.

B: Row space basis = both rows = $(1, 2, 4)$ and $(2, 5, 8)$; column space basis = two columns = $(1, 2)$ and $(2, 5)$; nullspace $(-4, 0, 1)$; left nullspace basis is empty because the space contains only $y = 0$.

3.6-5. $A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix}$ has those rows spanning its row space; $B = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$ has the same rows spanning its nullspace and $BA^T = 0$.

3.6-7. Invertible 3 by 3 matrix A: row space basis = column space basis = $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$; nullspace basis and left nullspace basis are empty. Matrix $B = \begin{bmatrix} A & A \end{bmatrix}$: row space basis $(1, 0, 0, 1, 0, 0)$, $(0, 1, 0, 0, 1, 0)$, and $(0, 0, 1, 0, 0, 1)$; column space basis $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$; nullspace basis $(-1, 0, 0, 1, 0, 0)$ and $(0, -1, 0, 0, 1, 0)$ and $(0, 0, -1, 0, 0, 1)$; left nullspace basis is empty.

3.6-28. B and C (checkers and chess) both have rank 2 if $p \neq 0$. Row 1 and row 2 are a basis for the row space of C, $B^T y = 0$ has 6 special solutions with -1 and 1 separated by a zero; $N(C^T)$ has $(-1, 0, 0, 0, 0, 0, 0, 1)$ and $(0, -1, 0, 0, 0, 0, 1, 0)$ and columns $3, 4, 5, 6$ of the identity I; $N(C)$ is a challenge.