The sixth week continues the work from chapter 3. Here’s the list of problems, followed by problem notes and a few answers.

**Section 3.3.** Exercises 1, 12, 21

**Section 3.4.** Exercises 1, 3, 4, 6, 13, 16, 33

**Problem Notes**

Issues for Strang’s problems will be communicated here. If there is a difficulty or impasse, then please send email, call 581-6879, or visit JWB 113.

**Some Answers**

**3.3.** Exercises 1, 21 have a textbook answer.

**3.3-12.** Invertible $r$ by $r$ submatrices use pivot rows and columns $S = \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix}$ and $S = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

**3.4.** Exercises 4, 6, 13, 16 have a textbook answer.

**3.4-1.** Row reduce the augmented matrix to upper triangular form

\[
\begin{pmatrix}
2 & 4 & 6 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & b_3 + b_2 - 2b_1
\end{pmatrix}
\]

Then $Ax = b$ has a solution when the last row is all zeros. This is the plane given by the equation $b_3 + b_2 - 2b_1 = 0$. The nullspace is obtained by solving $Ax = 0$, which is a step away by back-substitution.

The answer is $\vec{x}_{nullspace} = c_1 \vec{s}_1 + c_2 \vec{s}_2$ where $\vec{s}_1 = \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$, $\vec{s}_2 = \begin{pmatrix} 2 \\ -2 \\ 0 \\ 1 \end{pmatrix}$. Then the complete solution is

\[
c_1 \vec{s}_1 + c_2 \vec{s}_2 + \begin{pmatrix} b_1 \\ b_2 - b_1 \\ 0 \end{pmatrix}, \text{ subject to the restraint } b_3 + b_2 - 2b_1 = 0 \ (b_1, b_2 \text{ unrestrained}). \]

Choosing $b_1 = 4$ and $b_2 = 3$ with $c_1 = c_2 = 0$ gives particular solution $\vec{x}_{\text{particular}} = \begin{pmatrix} 4 \\ -1 \\ 0 \\ 0 \end{pmatrix}$.

**3.4-3.** $\vec{x}_{\text{complete}} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} + x_2 \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$. The matrix is singular but the equations are still solvable; $b$ is in the column space. Our particular solution has free variable $y = 0$.

**3.4-33.** If the complete solution to $Ax = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ is $x = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix}$ then $A = \begin{pmatrix} 1 & 0 \\ 3 & 0 \end{pmatrix}$. 