Linear Algebra 2270-2
Due in Week 5

The fifth week finishes the work from chapter 2 and starts chapter 3, which is full of topics. Here's the list of problems, followed by problem notes and a few answers.

Section 2.7. Exercises 2, 7, 20, 40

Section 3.1. Exercises 1, 3, 11, 12, 17, 18, 22, 23

Section 3.2. Exercises 4, 10, 15, 17, 18, 19, 24, 37

Problem Notes

Issues for Strang's problems will be communicated here. If there is a difficulty or impasse, then please send email, call 581-6879, or visit JWB 113.

Some Answers

2.7. Exercises 2, 20 are in the textbook.

2.7-7. (a) False, (b) False, (c) True, (d) True. Supply reasons and proofs.

2.7-40. Start from $Q^T Q = I$, as in $\begin{pmatrix} q_1^T \\ q_2^T \end{pmatrix} \begin{pmatrix} q_1, q_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(a) The diagonal entries give $q_1^T q_1 = 1$ and $q_2^T q_2 = 1$, unit vectors.

(b) The off-diagonal entry is $q_1^T q_2 = 0$ (and in general $q_i^T q_j = 0$).

(c) The leading example for Q is the rotation matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

3.1. Exercises 1, 3, 11, 23 are in the textbook.

3.1-12. For the plane $x + y - 2z = 4$, the sum of $(4, 0, 0)$ and $(0, 4, 0)$ is not on the plane. (The key is that this plane does not go through $(0, 0, 0)$.)

3.1-17. (a) The invertible matrices do not include the zero matrix, so they are not a subspace

(b) The sum of singular matrices $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ is not singular: not a subspace.

3.1-18. (a) True: The symmetric matrices do form a subspace (b) True: The matrices with $A^T = -A$ do form a subspace (c) False: The sum of two unsymmetrical matrices could be symmetric.

3.1-22. (a) Solution for every b (b) Solvable only if $b_3 = 0$. (c) Solvable only if $b_3 = b_2$.

3.2. Exercises 4, 15, 24, 37 are in the textbook.

3.2-10. (a) Impossible row 1 (b) A = invertible (c) A = all ones (d) $A = 2I$, $R = I$.

3.2-17. $A = \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$ gives the plane $x - 3y - z = 12$; y and z are free variables. The special solutions are $(3, 1, 0)$ and $(1, 0, 1)$.

3.2-18. Fill in 12 then 4 then 1 to get the complete solution to $x - 3y - z = 12$:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 12 \\ 0 \\ 0 \end{pmatrix} + y \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = x_{\text{particular}} + x_{\text{nullspace}}$$

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3.2-19. If $LUx = 0$, multiply by L^{-1} to find $Ux = 0$. Then U and LU have the same nullspace.