Name ___________________________ Class Time _____

Math 2270 Extra Credit Problems
Chapter 3
December 2011

These problems were created for Bretscher's textbook, but apply for Strang's book, except for the division by chapter. To find the background for a problem, consult Bretscher's textbook, which can be checked out from the math library or the LCB Math Center.

Due date: See the internet due dates. Records are locked on that date and only corrected, never appended.

Submitted work. Please submit one stapled package. Kindly label problems **Extra Credit**. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

Problem XC3.1-12. (Image and kernel)

For the matrix C below, display a frame sequence from C to $\mathbf{rref}(C)$. Write the image of C as the span of the pivot columns of C. Write the kernel of C as the list of partial derivatives $\partial\mathbf{x}/\partial c_1$, etc, where $\mathbf{x}$ is the vector general solution to $C\mathbf{x} = \mathbf{0}$.

$$C = \begin{pmatrix} 1 & -1 & -1 & 1 & 1 \\ -1 & 1 & 0 & 3 & 1 \\ 1 & -1 & -2 & 0 & 4 \\ 1 & -1 & -3 & 4 & 6 \end{pmatrix}$$

Problem XC3.1-22. (Geometry of a linear transformation)

Give an example of a 3×3 matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ has image equal to the plane through the three points $(0, 0, 0)$, $(0, 1, 1)$, $(1, 1, 0)$.

Problem XC3.1-38. (Image and kernel)

Express the image of the matrix A as the kernel of a matrix B. The matrix B can be a different size than A.

$$A = \begin{pmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 0 & 3 \\ 1 & -1 & -2 & 0 \\ 1 & -1 & -3 & 4 \end{pmatrix}$$

Problem XC3.1-50. (Kernel and image)

Let A be an $n \times n$ matrix and $B = \mathbf{rref}(A)$. Do A and B have the same kernels and images? Prove each assertion or give a counterexample.

Problem XC3.2-18. (Redundant vectors)

Identify the redundant vectors in the list, by application of the pivot theorem.

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 12 \\ 0 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 3 \\ 0 \\ 1 \end{pmatrix}.$$

Problem XC3.2-22. (Pivot columns)

Express the non-pivot columns of A as linear combinations of the pivot columns of A.

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

Problem XC3.2-46. (Basis for $\mathbf{ker}(A)$)

Find a basis for the kernel of A, using frame sequence methods.

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & -1 & 2 & 3 \end{pmatrix}$$

Problem XC3.2-48. (Independence with symbols)

Determine all values of the symbols a and b such that the following vectors are linearly independent.

$$\begin{pmatrix} a \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} a \\ 2b \\ 0 \\ 2b \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 3b \\ 0 \\ a-b \end{pmatrix}.$$

Problem XC3.3-10. (Basis for $\mathbf{ker}(A)$ and $\mathbf{dim}(A)$)

Find redundant columns by inspection and then find a basis for $\mathbf{dim}(A)$ and $\mathbf{ker}(A)$.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 2 & 2 & 2 \end{pmatrix}$$

Problem XC3.3-24. (Basis for $\mathbf{ker}(A)$ and $\mathbf{dim}(A)$)

Find $\mathbf{rref}(A)$ and then a basis for $\mathbf{ker}(A)$ and $\mathbf{dim}(A)$.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

Problem XC3.3-52. (Row space basis)

Find a basis for the row space of A consisting of columns of A^T.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 3 & 3 & 1 & 3 \\ 5 & 3 & 1 & 3 \end{pmatrix}$$

Problem XC3.3-64. (Kernels of matrices)

Prove or disprove that $\mathbf{ker}(A) = \mathbf{ker}(B)$.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 3 & 3 & 1 & 3 & 0 \\ 5 & 3 & 1 & 3 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 3 & 3 & 1 & 3 & 0 \\ 5 & 3 & 1 & 3 & 0 \end{pmatrix}.$$

Problem XC3.4-18. (Coordinates and spanning sets)

Let $V = \mathbf{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where the vectors are displayed below. Test for $\mathbf{x}$ in V, and if true, then report the coordinates of $\mathbf{x}$ relative to the the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$.

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} 7 \\ 1 \\ 3 \end{pmatrix}.$$

Problem XC3.4-30. (Matrix of a linear transformation)

Find the matrix B of the linear transformation T relative to the basis $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$.

$$T(\mathbf{x}) = \begin{pmatrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -2 & -2 & 8 \end{pmatrix},$$

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}.$$

Problem XC3.4-46. (Basis of a plane)

Consider the plane $2x_1 - 3x_2 + 4x_3 = 0$. Choose a basis $\mathbf{v}_1$, $\mathbf{v}_2$ for the plane, arbitrarily, your choice. Then determine the vector $\mathbf{x}$ which has coordinates 2, -1 relative to this basis.

End of extra credit problems chapter 3.