These problems were created for Bretscher’s textbook, but apply for Strang’s book, except for the division by chapter. To find the background for a problem, consult Bretscher’s textbook, which can be checked out from the math library or the LCB Math Center.

**Due date:** See the internet due dates. Records are locked on that date and only corrected, never appended.

**Submitted work.** Please submit one stapled package. Kindly label problems. Label each problem with its corresponding problem number. You may attach this printed sheet to simplify your work.

**Problem XC1.1-26. (Three possibilities)**

Determine which values of \( k \) correspond to (a) a unique solution, (b) no solution or (c) infinitely many solutions.

\[
\begin{align*}
x + 2y + kz &= 0 \\
2x + 4y + (k+1)z &= 2 \\
3x + 6y + (2k+1)z &= 2
\end{align*}
\]

**Problem XC1.1-30. (Polynomial interpolation)**

Find the polynomial \( f(x) = a + bx + cx^2 \) which passes through the points \((1, 9), (2, 24), (3, 47)\).

**Problem XC1.1-32. (Polynomial interpolation)**

Find all polynomials \( f(x) = a + bx + cx^2 \) which pass through the points \((1, 10), (2, 28)\) and \( f'(3) = 33 \).

**Problem XC1.2-22. (RREF)**

Report five types of \(3 \times 4\) matrices in RREF form.

**Problem XC1.2-28. (Combo rule)**

Consider the following systems.

\[
\begin{align*}
a_{11}x_1 + \cdots + a_{1n}x_n &= b_1 \\
&\vdots \\
a_{m1}x_1 + \cdots + a_{mn}x_n &= b_m
\end{align*}
\]

\[
\begin{align*}
a_{11}x_1 + \cdots + a_{1n}x_n &= b_1 \\
&\vdots \\
c_{k1}x_1 + \cdots + c_{kn}x_n &= d_k \\
&\vdots \\
a_{m1}x_1 + \cdots + a_{mn}x_n &= b_m
\end{align*}
\]

We assume the systems identical except for equation \(k\), which is obtained in the second system by applying a combination rule \( \text{combo}(r,k,c) \) to the first system. We assume \( r \neq k \) and then the coefficients in the second system are given by

\[
c_{kj} = a_{kj} + ca_{rj}, \quad j = 1, \ldots, n, \quad d_k = b_k + cb_r.
\]

(a) Prove that every solution of system (1) is a solution of system (2).

(b) Prove that every solution of system (2) is a solution of system (1).

**Problem XC1.2-30. (Polynomial interpolation)**

Find the polynomial \( f(x) = a + bx + cx^2 + dx^3 \) which satisfies \( f(1) = 8, f(2) = 24, f(3) = 24, f(4) = 110 \).
Problem XC1.3-26. (Matrix algebra)

Find a $3 \times 3$ matrix $A$ which satisfies the following relations.

$$A \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

End of extra credit problems chapter 1.