MATH 2270-003 **Sample Exam 2** Fall 2010

**1. (5 points)** Give a basis for each of the four fundamental subspaces associated to the following matrix:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

**2. (5 points)** $V$ is the span of the given vectors in $\mathbb{R}^4$. Find orthonormal vectors whose span is $V$.

$$\bar{v}_1 = \begin{pmatrix} 3 \\ 0 \\ 4 \\ 0 \end{pmatrix}, \bar{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$$

**3. (5 points)** For the subspace $V$ in the previous problem, give the matrix that projects $\mathbb{R}^4$ to $V$ and the matrix that projects $\mathbb{R}^4$ to $V^\perp$.

**4. (5 points)** Find the least squares best fit line for the points $(0, 1)$, $(2, 3)$, $(4, 4)$.

**5. (5 points)** Find the determinant of the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

**6. (5 points)** For the following matrix, find the eigenvalues and the maximum number of linearly independent eigenvectors. Find this many linearly independent eigenvectors.

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$$

**7. (5 points)** For the following matrix, find the eigenvalues and the maximum number of linearly independent eigenvectors. Find this many linearly independent eigenvectors.

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

**8. (5 points)** Describe the plane in $\mathbb{R}^3$ that contains the three points $(1, 0, 0)$, $(1, 1, 1)$, $(1, 2, 0)$.

**9. (5 points)** Suppose an $n \times n$ matrix $A$ has all eigenvalues equal to 0. Show that $A^n$ has all entries equal to 0.

**10. (1000000 points)** Prove the Cayley-Hamilton Theorem for matrices with real eigenvalues. You may assume the Jordan Form Theorem.

No new questions beyond this point.