MATH 2270-2 Exam 2 S2012

NAME (please print): ______________________________

No books or notes. No electronic devices, please.

These problems have credits 10 to 25, which is an estimate of the time required to write the solution.

1. (15 points) Let $A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 2 \end{pmatrix}$. Find a basis of vectors for each of the four fundamental subspaces, which are the nullspaces of A, A^T and the column spaces of A, A^T.

2. (25 points) Assume $V = \mathbf{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3)$ with $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

Find the Gram-Schmidt orthonormal vectors $\vec{q}_1, \vec{q}_2, \vec{q}_3$ whose span equals V.

3. (15 points) Find the least squares best fit line $y = v_1 x + v_2$ for the points $(1, 1)$, $(2, 3)$, $(3, 1)$, $(4, 4)$.

4. (20 points) Let $A = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$. Find all eigenpairs.

5. (15 points) Prove the Cayley-Hamilton Theorem for 2×2 matrices with real eigenvalues.

6. (10 points) How many eigenpairs for $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$?

No new questions beyond this point.