Math 2250 Maple Project 9, S2012. Tacoma Narrows.

NAME ______________________ CLASSTIME ________ VERSION A-K or L-Z

Circle the version - see problem L9.1. There are two (2) problems in this project. Please answer the questions A, B, C , ... associated with each problem. The original worksheet "2250mapleL9-S2012.mws" is a template for the solution; you must fill in the code and all comments. Sample code can be copied with the mouse. Use pencil freely to annotate the worksheet and to clarify the code and figures.

The problem headers for the S2012 revision of David Eyre's project (original was year 2000).

__________L9.1. NONLINEAR MCKENNA MODELS

__________L9.2. MCKENNA NON-HOOKES LAW CABLE MODEL.

L9.1. PROBLEM (NONLINEAR MCKENNA MODELS)

There are three (3) parts L9.1A to L9.1C to complete. Mostly, this is mouse copying. Retyping the maple code by hand is not recommended.

NONLINEAR TORSIONAL MODEL WITH GEOMETRY INCLUDED.
Consider the nonlinear, forced, damped oscillator equation for torsional motion, with bridge geometry included,

```
    x'' + 0.05 x' + 2.4 sin(x)cos(x) = 0.06 cos (12 t/10) ,
    x(0) = x0,  x'(0) = v0
```

and its corresponding linearized equation

```
   x'' + 0.05 x' + 2.4 x = 0.06 cos (12 t/10) ,
   x(0) = x0,  x'(0) = v0.
```

The spring-mass system parameters are m=1, c = 0.05, k = 2.4, w = 1.2 , F = 0.06. Maple code used to solve and plot the solutions appears below.

```
   # Use "copy as maple text" for maple 6+.
   x0:=0: a:=200: b:=300:  # For part A. Change it for part B!
   v0:=0: m:=1:  F := 0.06:  w := 1.2: m:=1: c:= 0.05: k:= 2.4:
   with(DEtools):  opts:=stepsize=0.1:
   deLinear:= m*diff(x(t),t,t) + c*diff(x(t),t) + k*x(t) = F*cos(w*t):
   IClinear:=[[x(0)=x0,D(x)(0)=v0]]:
   DEplot(deLinear,x(t),t=a..b,IClinear,opts,title='Linear');
   deNonLinear:= m*diff(x(t),t,t) + c*diff(x(t),t) +
                 k*sin(x(t))*cos(x(t)) = F*cos(w*t):
   ICnonlinear:=[[x(0)=x0,D(x)(0)=v0]]:
   DEplot(deNonLinear,x(t),t=a..b,ICnonlinear,opts,title='NonLinear');
```

9.1A. Let x0=0, v0=0. Plot the solutions of the linear and nonlinear equations from t=200 to t=300. These plots represent the steady state solutions of the two equations.

9.1B. Let x0=1.2, v0=0. Plot the solutions of the linear and

```
          nonlinear equations from t=220 to t=320. These plots represent
          the steady state solutions of the two equation, with new
          starting value x0=1.2. [You must modify line 1 of the maple
          code!]

          The two linear plots in A and B have to be identical to the
          plot of xss(t). The reason is the superposition formula (see
          E&P) x(t)=xh(t)+xss(t), even though the homogeneous solution
          xh(t) is different for the two plots.  This is because xh(t)
          has limit zero at t=infinity.

    9.1C. Determine the ratio of the apparent amplitudes (a number > 1)
          for the nonlinear plots in A and B. Do "large sustained
          oscillations" appear in the plot of the nonlinear
          steady-state?
>
> #L9.1-A
> #L9.1-B
> #L9.1-C
>
```

L9.2. PROBLEM (MCKENNA'S NON-HOOKE'S LAW CABLE MODEL)

There are three (3) parts L9.2A to L9.2C to complete. Mostly, this is mouse copying. Retyping the maple code by hand is not recommended.

The model of McKenna studies the bridge with a nonlinear, forced, damped oscillator equation for torsional motion that accounts for the non-Hooke's law cables coupled to the equations for vertical motion. The equations in this case couple the torsional motion with the vertical motion. The equations are:

```
    x'' + c x' - k G(x,y) = F sin wt,   x(0) = x0,  x'(0) = x1,
    y'' + c y' + (k/3) H(x,y) = g ,     y(0) = y0,  y'(0) = y1,
```

where x(t) is the torsional motion and y(t) is the vertical motion. The functions G(x,y) and H(x,y) are the models of the force generated by the cable when it is contracted and stretched. Below is sample code for writing the differential equations and for plotting the solutions. It is ready to copy with the mouse.

```
with(DEtools):
w := 1.3:  F := 0.05:  f(t) := F*sin(w*t):
c := 0.01:  k1 := 0.2:  k2 := 0.4:  g := 9.8:  L := 6:
STEP:=x->piecewise(x<0,0,1):
fp(t) := y(t)+(L*sin(x(t))):
fm(t) := y(t)-(L*sin(x(t))):
Sm(t) := STEP(fm(t))*fm(t):
Sp(t) := STEP(fp(t))*fp(t):
sys := {
     diff(x(t),t,t) + c*diff(x(t),t) - k1*cos(x(t))*(Sm(t)-Sp(t))=f(t),
     diff(y(t),t,t) + c*diff(y(t),t) + k2*(Sm(t)+Sp(t)) = g}:
ic := [[x(0)=0, D(x)(0)=0, y(0)=27.25, D(y)(0)=0]]:
vars:=[x(t),y(t)]:
opts:=stepsize=0.1:
```

```
DEplot(sys,vars,t=0..300,ic,opts,scene=[t,x]);
```

The amazing thing that happens in this simulation is that the large vertical oscillations take all the tension out of the springs and they induce large torsional oscillations.

L9.2A. TORSIONAL OSCILLATION PLOT. Get the sample code above to produce the plot of x(t) [that's what scene=[t,x] means].

L9.2B. ROADWAY TILT ANGLE. Estimate the number of degrees the roadway tilts based on the plot. Recall that x in the plot is reported in radians. Comment on the agreement of this result with historical data and the video evidence in the film clip.

Tip: Average the five largest amplitudes in the plot to find an average maximum amplitude for t=0 to t=300. Convert to degrees using Pi radians = 180 degrees. The film clip shows roadway maximum tilt of 30 to 45 degrees, approximately.

L9.2C. VERTICAL OSCILLATION PLOT. Modify the DEplot code to scene=[t,y] and plot the oscillation y(t) on t=0 to t=300. The plot is supposed to show 30-foot vertical oscillations along the roadway that dampen to 7-foot vertical oscillations after 300 seconds.

The agreement between these oscillation results and the historical data for Tacoma Narrows, especially the visual data present in the film clip of the bridge disaster, should be clear from the plots. This is your only answer check for the plot results.

```
>
> #L9.2-A Torsional plot t-versus-x
> #L9.2-b Roadway tilt angle estimate in degrees + comments.
> #L9.2-C Vertical plot t-versus-y.
>
```