Math 2250 Extra Credit Problems
Chapter 1
S2012

Due date: The due date for these problems is week 13. Records are locked on that date and only corrected, never appended. These problems can replace any missed problem from chapter 1. The maximum that can be earned for chapter 1 is 100%. Credits earned for these problems cannot be transferred to any other chapter.

Submitted work. Please submit one stapled package per problem. Kindly label problems $\boxed{\textbf{Extra Credit}}$. Label each problem with its corresponding problem number, e.g., $\boxed{\text{Xc1.2-4}}$. You may attach this printed sheet to simplify your work.

Problem Xc1.2-4. (**Quadrature**)
Solve $y' = x^{-2} + x^{-1}$, $y(1) = 2$.

Problem Xc1.2-10. (**Quadrature**)
Solve $y' = xe^{-2x} + x^2$, $y(0) = 2$.

Problem Xc1.3-8. (**Picard's theorem**)
Find a box with center $x = 0$, $y = 0$ to which Picard's theorem applies, verifying also continuity of $f(x, y)$ and $f_y(x, y)$ in the box, for the equation
$$y' = x\sqrt{x + y + 1}, \quad y(0) = 0.$$

Problem Xc1.3-14. (**Peano's theorem**)
Does Peano's theorem apply to establish existence of at least one solution, for the problem below? Please carefully check the hypothesis of the theorem, which is continuity of $f(x, y)$ on a box with center $x = 0$, $y = 1$.
$$y' = 3(y - 1)^{1/3}, \quad y(0) = 1.$$

Problem Xc1.4-6. (**Separable DE**)
Solve for equilibrium and non-equilibrium solutions (find the general solution).
$$y' = 2x \sec y.$$

Problem Xc1.4-17. (**Separability test**)
Use the test to verify that the equation $y' = e^x + e^y$ is not separable.

Problem Xc1.4-18. (**Separability test**)
Find a factorization $f(x, y) = F(x)G(y)$ for the problem below and then determine all non-equilibrium solutions.
$$y' = x^2(y^2 + y) + y^2 + x^2y + 2y + x^2 + 1.$$

Problem Xc1.4-49. (**Newton cooling**)
A roast is put into an oven whose temperature is 400° F. The meat thermometer was initially at 40° F and after 30 minutes it rose to 90° F. The roast is done when the thermometer reaches 340° F. How long does it take to cook the roast?

Problem Xc1.5-4. (**Linear DE**)
Solve $y' - 2xy = e^{x^2}$.

Problem Xc1.5-16. (**Linear DE**)

Solve $y' = (1 - y) \cos x$, $y(\pi) = 2$.

Problem Xc1.5-24. (**Linear DE**)

Solve $(x^2 + 4)y' + 3xy = x$, $y(0) = 1$.

Problem Xc1.5-38. (**Brine tank**)

Solve the brine tank problem

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\begin{aligned}
x'(t) &= -4x(t), \\
y'(t) &= 4x(t) - 5y(t).
\end{aligned}
$$

End of extra credit problems chapter 1.