Name ____________________ Class Time _____

Math 2250 Extra Credit Problems Chapter 10X
Challenging Laplace Applications S2012

Due date: Submit these problems by the deadline for semester extra credit, usually the day after classes end, or as posted on the due dates page at the course web site. Records are locked on that date and only corrected, never appended. The scores on Ch10X extra credit can replace *any* missing score for the entire semester.

Submitted work. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the problems $\boxed{\textbf{Extra Credit}}$. Label each problem with its corresponding problem number, e.g., $\boxed{\text{Xc10X.3-20}}$.

Problem Xc10X.1. (Inverse transform)

Solve for $f(t)$, given $\mathcal{L}(f(t)) = e^{-2s}\dfrac{s}{(s+1)(s^2+1)}$.

Problem Xc10X.2. (Inverse transform)

Solve for $f(t)$, given $\mathcal{L}(f(t)) = \dfrac{d}{ds}\left(e^{-2s}\dfrac{s+1}{s^2(s^2+1)}\right)$.

Problem Xc10X.3. (Inverse transform)

Solve for $f(t)$, given $\mathcal{L}(f(t)) = \dfrac{s+4}{s^2(s^2+2s+2)} + \dfrac{e^{-2s}}{s(1-e^{-2s})}$. Hint: Look on the inside cover of Edwards-Penney.

Problem Xc10X.4. (Resolvent Equation and e^{At})

Leverrier and Faddeeva derived the following recursions for the coefficients in the expansion

$$(sI - A)^{-1} = \sum_{k=0}^{n-1} \frac{s^{n-k-1}}{\det(sI - A)} A_k.$$

$$\det(sI - A) = s^n - \sum_{k=0}^{n-1} c_k s^k, \quad A_0 = I, \quad A_k = A_{k-1}A - c_{n-k}I.$$

Then

$$e^{At} = \mathcal{L}^{-1}\left((sI - A)^{-1}\right) = \sum_{k=0}^{n-1} \mathcal{L}^{-1}\left(\frac{s^{n-k-1}}{\det(sI - A)}\right) A_k.$$

(a) Write out A_0, A_1 for a 2×2 matrix A, given $\det(sI - A) = s^2 - c_1 s - c_0$.
(b) Write out A_0, A_1, A_2 for a 3×3 matrix A, given $\det(sI - A) = s^3 - c_2 s^2 - c_2 s - c_0$.
(c) Apply the Leverrier-Faddeeva formulas to find the matrix exponential e^{At} for the special case

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

(d) Check the answer for e^{At} in a computer algebra system.

Problem Xc10X.5. (Laplace transform)

Find $\mathcal{L}(f(t))$, given $f(t) = \dfrac{\sinh(t)}{t}$.

Problem Xc10X.6. (Laplace transform)

Find $\mathcal{L}(f(t))$, given $f(t) = \dfrac{d}{dt}(\sinh(t)\cosh(t))$.

Problem Xc10X.7. (Laplace's method)

Solve by Laplace's method the initial value problem $tx''(t) - 2x'(t) + tx(t) = 0$, $x(0) = 1$, $x'(0) = 0$. Check the answer in maple.

Problem Xc10X.8. (Laplace's method)

Solve by Laplace's method the initial value problem $tx''(t)+2x'(t)+x(t) = \delta(t)+\delta(t-1)+\delta(t-2)$, $x(0) = 0$, $x'(0) = 1$. Check the answer in maple using `dsolve({de,ic},x(t),method=laplace)`.

Problem Xc10X.9. (Backward table)

Solve for $f(t)$, given $F(s) = \mathcal{L}(f(t))$.

$$F(s) = \frac{5s^4 + 16s^3 + 2560s^2 + 1600s + 327680}{(s^2 + 100)(s^2 + 256)^2}.$$

Problem Xc10X.10. (Backward table)

Solve for $f(t)$, given $F(s) = \mathcal{L}(f(t))$.

$$F(s) = \frac{5s^4 - 24s^3 + 80s^2 - 32s + 256}{(s - 2)^3(s^2 + 16)^2}.$$

End of extra credit problems chapter 10X.