

**Math 2250 Maple Project 3: Numerical Methods
S2010**

Due date: See the internet due dates. Maple lab 3 has three problems L3.1, L3.2, L3.3.

References: Code in maple appears in 2250mapleL3-S2010.txt at URL <http://www.math.utah.edu/~gustafso/>. This document: 2250mapleL3-S2010.pdf. Other related and required documents are available at the web site:

- Numerical Solution of First Order DE (typeset, 19 pages, 220k pdf). A resource similar to the textbook, with maple examples and deeper detail. It is for a second reading, in case Edwards-Penney left too many questions unanswered.
- Sample Report for 2.4-3 (pdf 3 pages, 350k). This outline might be useful, if you are confused about which details to include.
- Numerical DE coding hints, TEXT Document (1 page, 2k). This document is appended here, for completeness. The web copy 2250mapleL3-S2010.txt is suited for mouse copying.
- Sample maple code for Euler, Heun, RK4 (maple worksheet). Use 2250mapleL3-S2010-snips.mws to load maple sample code without mouse copying.
- Sample maple code for exact/error reporting (maple worksheet). Normally not useful, because a hand calculator can do it faster.

Problem Exam Review ER-1. (E & P Exercise 2.4-6, Symbolic Solution)

The symbolic solution of $y' = -2xy$, $y(0) = 2$ is $y = 2e^{-x^2}$. Using methods from the textbook, Chapter 1, display the details of the derivation for this symbolic solution, plus a full answer check.

The answer is used in the problems of maple Lab 3, in order to check the calculations.

This problem was submitted earlier, before Exam 1.

This problem was not submitted earlier. The solution will be submitted with maple lab 3.

Staple this page on top of your hand-written report

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Problem L3.1. (E & P Exercise 2.4-6)

Consider the initial value problem $y' = -2xy$, $y(0) = 2$ with symbolic solution $y = 2e^{-x^2}$. Apply Euler's method to produce two dot tables, as follows. The first has three rows, $h = 0.25$. The second has six rows, $h = 0.1$. Reproduce the table below and fill in missing digits. Follow the sample report for Edwards-Penney Exercise 2.4-3:

<http://www.math.utah.edu/~gustafso/2250SampleProblem2.4-3.pdf>

Reference Exam Review ER-1 for the symbolic solution. Hand written work includes a check of the computer answer for $y(x_0 + h)$, that is, $y(0.25)$ for the program using $h = 0.25$ and $y(0.1)$ for the program using $h = 0.1$. The check makes sure the computer program obtains the correct answer for the very first step.

h	actual $y(.5)$	approx $y(.5)$
0.25	1.558	1.750?????
0.1	1.558	1.627?076?

Problem L3.2. (E & P Exercise 2.5-6)

Consider the initial value problem $y' = -2xy$, $y(0) = 2$ with symbolic solution $y = 2e^{-x^2}$. Apply Heun's method (Improved Euler) to produce one dot table of six rows, $h = 0.1$. Reproduce the table below and fill in missing digits. Follow the sample report for Exercise 2.4-3 as in problem L3.1 above. Hand written work includes a check of the computer answer for $y(x_0 + h)$, that is, $y(0.1)$ for the program using $h = 0.1$.

x	0.0	0.1	0.2	0.3	0.4	0.5
actual $y(x)$	2.000000000	1.980099667	1.921578878	1.827862371	1.704287578	1.557601566
approx $y(x)$	2.0000000	1.980????	1.921????	1.827????	1.704????	1.557????

Problem L3.3. (E & P Exercise 2.6-6)

Consider the initial value problem $y' = -2xy$, $y(0) = 2$ with symbolic solution $y = 2e^{-x^2}$. Apply the RK4 method to produce one dot table of three rows, $h = 0.25$. Reproduce the table below, filling in the missing digits. Follow the sample report for Exercise 2.4-3 as in problem L3.1 above. Forget hand calculator checks, because the table suggests that comparison with the symbolic solution is enough.

x	0.00	0.25	0.50
actual $y(x)$	2.000000000	1.878826126	1.557601566
approx $y(x)$	2.000000000	1.8788????0	1.557??329?

Staple this page on top of your hand-written and maple worksheet report

```
#Dr. Gustafson,
#I am in your 2250-4 class and am continually struggling with the codes for
#the problems in section 2.4, 2.5, and 2.6. Actually 2.5 (Improved Euler)
#seems to be going pretty well, but I can't get very good results for 2.4
#(Euler) or 2.6 (Runge-Kutta Idea). Is there a website where I may be able
#to find help and/or codes on these sections, as nothing I type in for my
#codes will seem to work? Thanks for your time.
```

```
# =====
# Can't copy with the mouse? A work-around:
# =====
# Run the application "xclipboard &" to capture the mouse copies of this
# file. Keep xclipboard near the xmaple window. Go to the mozilla firefox
# window, copy with the mouse. Switch to the xclipboard window. Copy
# with the mouse from the xclipboard. Then paste with mouse button 2 or
# mouse button 3 into xmaple.
```

```
# =====
# Maple code doesn't work? Read this:
# =====
# To type in a group, hold shift then press return, except
# for the last line of group, in which case use just return.
# If you copy multiple groups with the mouse, then split
# them using key F3 with the cursor placed at the front of
# a line where the split is to happen.
```

```
# Warning: These snips of code made for  $y'=1-x-y$ ,  $y(0)=3$ .
# Code computes approx values for  $y(0.1)$  to  $y(1.0)$ .
# 'Dots' is the list of dots for connect-the-dots graphics.
```

```
# =====
# Euler. Group 1, initialize.
f:=(x,y)->1-x-y:
x0:=0:y0:=3:h:=0.1:Dots:=[x0,y0]:n:=10:
# Group 2, repeat n times. Euler's method
for i from 1 to n do
Y:=y0+h*f(x0,y0);
x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
od:
# Group 3, display dots and plot.
Dots;
plot([Dots]);
```

```
# =====
# Heun. Group 1, initialize.
f:=(x,y)->1-x-y:
x0:=0:y0:=3:h:=0.1:Dots:=[x0,y0]:n:=10:
# Group 2, repeat n times. Heun method.
for i from 1 to n do
Y1:=y0+h*f(x0,y0);
Y:=y0+h*(f(x0,y0)+f(x0+h,Y1))/2:
x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
od:
# Group 3, display dots and plot.
Dots;
plot([Dots]);
```

```
# =====
# RK4. Group 1, initialize.
f:=(x,y)->1-x-y:
x0:=0:y0:=3:h:=0.1:Dots:=[x0,y0]:n:=10:
```

```

# Group 2, repeat n times. RK4 method.
for i from 1 to n do
  k1:=h*f(x0,y0):
  k2:=h*f(x0+h/2,y0+k1/2):
  k3:=h*f(x0+h/2,y0+k2/2):
  k4:=h*f(x0+h,y0+k3):
  Y:=y0+(k1+2*k2+2*k3+k4)/6:
  x0:=x0+h:y0:=Y:Dots:=Dots,[x0,y0];
od:
# Group 3, display some dots and plot.
Dots[1],Dots[2],Dots[n+1];
plot([Dots]);

# Code snips for exact/error reports
# =====
# Making multiple curves on one plot
# =====
Exact:=x->2-x+exp(-x);          # An exact solution
plot({Exact(x),[Dots]},x=0..1/2); # plot exact and approx solutions

# =====
# How to create a Dots table for the exact solution
# =====
Exact:= x -> 2-x+exp(-x):n:=10:
ExactDots:=seq([Dots[j][1],Exact(Dots[j][1])],j=1..n+1);

# =====
# How to define and print percentage relative error:
# =====
P:=unapply(evalf(100*abs(exact-approx)/abs(exact)),(exact,approx));
ExactVal:=Exact(Dots[11][1]): # Compute exact y-value for x=1.0
ApproxVal:=Dots[11][2]:      # Get Euler approx y-value for x=1.0
P(ExactVal,ApproxVal);      # print percent relative error

# =====
# How to create a Dots table for percentage error
# =====
P:=unapply(evalf(100*abs(exact-approx)/abs(exact)),(exact,approx));
Pdots:=seq([Dots[j][1],P(Exact(Dots[j][1]),Dots[j][2])],j=1..11);

# =====
# Printing results and tables
# Make tables with a pencil, it saves time.
# =====
# To extract and print items 1,101,201,1001 from a list:
Dots1:=Dots[1],Dots[101],Dots[201],Dots[1001];

# =====
# Loop control
# =====
# To automate the production of a Dots list,
# enclose the desired code between 1 and 2 below.
# 1. for k from 1 to 10 do
# 2. od:
#
# Keyword "od:" is short for "end do:"
# Use ":" to stop loop results from printing.

# =====

```

```
# Debug
# =====
# To remove loop control and do it by hand, insert
# pound (#) signs as follows:
# 1. # for k from 1 to 10 do
# 2. # od:
# The hand-done loop is made by placing the mouse cursor
# in the group, then press return. Repeat for each loop step,
# which is 10 times for the loop above.
# the end
```

End of Maple Lab 3: Numerical Methods.