Name ________________________ Class Time _____

Math 2250 Maple Project 1
Spring 2010

Due date: See the internet due dates. Maple Project 1 has four problems L1.1, L1.2, L1.3, L1.4.

Submitted work. Please submit one stapled package. Some handwritten work is expected, which should display the steps shown in examples below. Maple code is expected to parallel the code given in the examples below. The final pages are appendices made by printing one or more maple work sheets.

Tutorials. Students without maple exposure should attend one of the maple tutorials held for 2250 in the second week of the semester. The brave may start their own tutorial by attempting this project unaided or by starting with the maple online tutorial; see Example 7 below.

References: Code in `maple` appears in `2250mapleL1-S2010.txt` at URL `http://www.math.utah.edu/~gustafso/s2010/`. This document: `2250mapleL1-S2010.pdf`.

Problem L1.1. (Quadratic equation)

Solve the quadratic equation $ax^2 + bx + c = 0$ and display its factorization:
(L1.1A) $a = 8$, $b = 22$, $c = 15$;
(L1.1B) $a = 1$, $b = 2$, $c = 6$;
(L1.1C) $a = 2$, $b = 16$, $c = 32$.
In your solution, show the **solution steps by hand** and also the `maple` code which checks the answer. Details should parallel those of Example 1, below.

Problem L1.2. (Functions and plotting)

Define the following functions and plot domains, then plot them.
(L1.2A) $e^{-x}\sin(5\pi x)$, $0 \le x \le 1$.
(L1.2B) $|4\ln(5 + x) - 1|$,$-2 \le x \le 4$.
(L1.2C) Let $a = 5$, $b = 8$, $c = \pi/12$, $t_0 = 15$. Plot $a + b\cos(c(t - t_0))$, $0 \le t \le 48$.
(L1.2D) $\sinh(2\sin(\theta) + 3\cos(\theta))$, $-\pi \le \theta \le \pi$.
In the four solutions, show only the maple code and the graphics. Please shrink the graphics to about 1/4 page, before printing. See Example 2 below.

Problem L1.3. (Derivatives)

Compute the indicated derivative(s) by hand and then check your answer in maple.

(L1.3A) $\dfrac{dy}{dx}$, $y = \dfrac{(x-1)^2}{x^3+1} + xe^{-2x}\cos(3x)$

(L1.3B) $\partial_x z$, $\partial_y z$, $z = \cosh(x)\sinh(x + y)$

(L1.3C) $\partial_{c_1} u$, $\partial_{c_2} u$, $\partial_{c_3} u$, $u = c_1x^2 + c_2 + c_3xe^{2x}$

(L1.3D) $\partial_x\partial_y w$, $w = e^{xy}\tan(x^2 + y^2 + 1)$

Problem L1.4. (Vector Partials and Jacobian)

(L1.4A) Compute the vector partials $\partial_{t_1}\mathbf{u}$ and $\partial_{t_2}\mathbf{u}$, for

$$\mathbf{u} = (1 + 2t_1 - t_2)\vec{\imath} + (2 - t_1 + 3t_2)\vec{\jmath} + (2t_1 + 5t_2)\vec{z}.$$

(L1.4B) Compute the Jacobian matrix and determinant at $x = y = 0$ for vector function $z_1\vec{\imath} + z_2\vec{\jmath}$, given

$$z_1 = x(x + 2y - 1), \quad z_2 = y(-x + 3y + 2).$$

Do a hand computation on paper and then check your answer in maple.

Staple this page on top of the maple work sheets. **Start the hand solution on a separate page.**
Examples on the next page . . .

Example 1. Solve $2x^2 + 8x + 12 = 0$ by hand and check using `maple`.

Solution: Divide by 2. Then square-completion $(x+2)^2 + 2 = 0$ gives conjugate roots $x = -2 + \sqrt{2}i$, $x = -2 - \sqrt{2}i$. By the root-factor theorem of college algebra, the quadratic equation has factors $(x - \text{root } 1)$ and $(x - \text{root } 2)$. Then the factorization is $2(x + 2 - \sqrt{2}i)(x + 2 + \sqrt{2}i) = 0$, because FOIL expansion gives leading term $2x^2$.

The roots may also be found from the quadratic formula, in which case the root and factor theorems of algebra apply to translate each root $x = r$ into a factor $x - r$ of the quadratic equation.

The maple check reproduces the original quadratic equation from its factors and leading coefficient. The `maple` code is

```
eq:=2*x^2+8*x+12:
ans:=[solve(eq=0,x)];
eq1:=2*(x-ans[1])*(x-ans[2])=0;
expand(eq1);
```

Notation: Square brackets delimit an array, e.g., `F:=[-1,3,5];` defines array `F` with three elements −1, 3, 5. Symbol `F[1]` extracts the first element from array `F`, while `F[2]` extracts the second element.

Example 2. Define a function $y = x^2 + 5x + 6$ on $-4 \le x \le -1$ using `maple` and plot it.

Solution: The `maple` code which applies is

```
f:=unapply(x^2+5*x+6,x): a:=-4: b:=-1:
plot(f(x),x=a..b);
```

The construct `f:=unapply(x^2+5*x+6,x)` is an inline function definition. Subsequent use of the symbol `f` requires two parentheses and a function argument, e.g., `f(x)`, `f(-1.1)` are valid.

The inline function definition `f:=x->x^2+5*x+6` uses a minus sign `-` and a greater than sign `>` to separate the variable name `x` from the function definition `x^2+5*x+6`. This alternative construction may appear in later maple code. In this elementary example, there is no difference between the two constructs. The maple function `unapply` is more robust and produces fewer surprises for novice maple users.

Constants and Empty Plots. The constant π is coded in `maple` as `Pi`, the upper and lowercase letters being significant. Coding `c=Pi;` prints an equation. Coding `c:=Pi;` assigns π to symbol `c`. The error message **empty plot** can mean that a symbol is undefined. For example, `plot(x+PI,x=0..1);` will not plot. To see why, use `p:=plot(x+PI,x=0..1);` to display the plot data. The offending undefined symbol is `PI`, which is a different symbol than `Pi` or `pi`.

Example 3. Compute the derivative of $y = \dfrac{3x^4}{(x+1)^2} + \sin 2x + e^x \cos 3x$ and check the answer in maple.

Solution: The hand solution uses the quotient rule, chain rule and product rule. Let $y = y_1 + y_2 + y_3$, where $y_1 = \dfrac{3x^4}{(x+1)^2}$, $y_2 = \sin 2x$ and $y_3 = e^x \cos 3x$. Then

$$\begin{aligned} y_1' &= \frac{(\text{top})'(\text{bot}) - (\text{top})(\text{bot})'}{(\text{bot})^2} &&= \frac{12x^2(x+1)^2 - 6x^2(x+1)}{(x+1)^4} \\ y_2' &= 2\cos 2x \text{ by the chain rule} \\ y_3' &= (\text{first})'(\text{second}) + (\text{first})(\text{second})' &&= e^x \cos 3x - 3e^x \sin 3x. \end{aligned}$$

The code for maple:

```
y1:=3*x^4/(x+1)^2: y2:=sin(2*x): y3:=exp(x)*\cos(3*x):
y1prime:=diff(y1,x);
y2prime:=diff(y2,x);
y3prime:=diff(y3,x);
yprime:=y1prime+y2prime+y3prime;
```

The displayed answer for y':

$$12\,\frac{x^3}{(x+1)^2} - 6\,\frac{x^4}{(x+1)^3} + 2\,\cos(2\,x) + e^x\cos(3\,x) - 3\,e^x\sin(3\,x)$$

Example 4. Given $z = 2y + xy + (3x+4y)^2$, compute $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ and check the answer.

Solution: By hand, using the chain rule, $\dfrac{\partial z}{\partial x} = 0 + y + 6(3x+4y)$ and $\dfrac{\partial z}{\partial y} = 2 + x + 8(3x+4y)$. The maple answer check:

```
z:=2*y+x*y+(3*x+4*y)^2:
dzdx:=diff(z,x);dzdy:=diff(z,y);
```

Example 5. Compute the Jacobian matrix $\begin{pmatrix} \partial_x z_1 & \partial_y z_1 \\ \partial_x z_2 & \partial_y z_2 \end{pmatrix}$ and its determinant $\begin{vmatrix} \partial_x z_1 & \partial_y z_1 \\ \partial_x z_2 & \partial_y z_2 \end{vmatrix}$ at $x = y = 0$:

$$z_1 = (3x+y+1)^2, \quad z_2 = xy + 2x + 3y$$

Solution: The hand solution:

$$\begin{array}{lllllll} a &=& \partial_x z_1(0,0) &=& 6(3x+y+1)|_{x=y=0} &=& 6, \\ b &=& \partial_y z_1(0,0) &=& 2(3x+y+1)|_{x=y=0} &=& 2, \\ c &=& \partial_x z_2(0,0) &=& (y+2+0)|_{x=y=0} &=& 2, \\ d &=& \partial_y z_2(0,0) &=& (x+0+3)|_{x=y=0} &=& 3. \end{array}$$

Then

$$\text{Jacobian}(0,0) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 2 & 3 \end{pmatrix},$$

$$\det(\text{Jacobian})(0,0)) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = 18 - 4 = 14.$$

The code for maple:

```
z1:=(3*x+y+1)^2: z2:=x*y+2*x+3*y:
J:=VectorCalculus[Jacobian]([z1,z2],[x,y]);
DJ:=LinearAlgebra[Determinant](J);
subs(x=0,y=0,J);
subs(x=0,y=0,DJ);
```

The answers from maple:

$$a = 18x + 6y + 6, \quad b = 6x + 2y + 2, \quad c = y + 2, \quad d = x + 3, \quad J = \begin{pmatrix} 18x + 6y + 6 & 6x + 2y + 2 \\ y + 2 & x + 3 \end{pmatrix},$$

$$DJ = 18x^2 + 48x + 12y + 14 - 2y^2, \quad J(0,0) = \begin{pmatrix} 6 & 2 \\ 2 & 3 \end{pmatrix}, \quad \det(J) = \begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix} = 14.$$

Example 6. Get `maple` help.

Solution: Get help on maple's *unapply* function by entering `?unapply` into a `maple` worksheet. The question mark precedes the maple keyword. All maple help has examples, normally at the end of the help sheet. Here are some keywords to try:

```
unapply, plot, expand, factor, ifactor, solve, fsolve, array, set, for, do, parse, plot3d,
dsolve, int, diff, newuser, exp, sin, cos, tan, sinh, cosh, ln, log
```

Example 7. Run the `maple` tutorial in `maple` versions 6 to 12.

Solution: In a `maple` worksheet, enter `?newuser` and choose the *New User's Tour*. In the tour, you will learn some basics of `maple`. See the web site links to other tutorials and reference cards.

End of Maple Lab 1.