2,4-3 (Symbolic Solution) Derive and verify the solution for

$$\begin{cases} y' = y+1, \\ y(0)=1. \end{cases}$$

Exact solution $y(x) = 2e^x - 1$

Derivation:

$y' - y = 1$ Standard linear form

$\frac{(e^{-x}y)'}{e^{-x}} = 1$ Replace LHS by integrating factor fraction $(Qy)'/Q$; $Q = e^{\int(-1)dx} = e^{-x}$.

$(e^{-x}y)' = e^{-x}$ Quadrature preparation.

$e^{-x}y = -e^{-x} + c$ Apply quadrature

$y = -1 + ce^x$ General solution

$y = -1 + 2e^x$ Evaluate $c=2$ from $y(0)=1$.

Ans Check: problem 2,4-3 p 119 E&P.