# MATHEMATICS 3150-1
# Partial Differential Equations
# Fall semester 2009

**Time**: Mon-Wed 11:50am, LCB 225

**Instructor**: Professor Grant B. Gustafson[1]

**Office Hours**: JWB 113, Mon-Wed before and after class [appointment appreciated]. Other times appear on my door card. From computers, read the door card link at the course web site.

**Telephone**: 801-581-6879 [113JWB]. Please use email whenever possible.

**Email**: `ggustaf@math.utah.edu`

**Web site**: `http://www.math.utah.edu/~gustafso/`

**Tutoring**: The Math Center, abbreviation MC, is located in the basement of building LCB. Free tutoring is offered Mon–Thu from 8 a.m. to 8 p.m., and from 8 a.m. to 6 p.m. on Friday. Some math tutors can field questions about Math 3150 topics. To see the times and specialities of various tutors, consult the web address

`www.math.utah.edu/ugrad/tutoring.html`.

**Texts**:

**Required Text**. *Partial Differential Equations and Boundary Value Problems* 2/E, by Nakhlé Asmar.

Publication info: Prentice Hall, hardcover 2nd edition (May 24, 2004), 816 pages, ISBN-10: 0131480960, ISBN-13: 978-0131480964. Amazon has sold it for $101.00 with free shipping. Biblio.com offers it new in hardcover for $47.00 and up. Don't buy the first edition (2000).

**Student Solution Manual**. The second edition PDF manual is available for free from Asmar's site. Maple notebooks on several topics are also available.

`http://www.math.missouri.edu/ñakhle/pdebvp/student-manual.pdf`

`http://www.math.missouri.edu/ nakhle/pdebvp/maple-notebooks.html`

**WWW Documents for 3150**. Browse the web site

`http://www.math.utah.edu/~gustafso/`.

All are pdf or text documents that can be printed from `Mozilla Firefox`, `MS-windows iexplorer`, `OS/X Safari` and other web browsers that support printing of text and `pdf` files. Author: G.B. Gustafson. The notes and slides may be freely viewed and printed.

## Prerequisites

Math 1210 and 1220 or the equivalent (Calculus I and II). A course in ordinary differential equations, 2250 or 2280. A course in linear algebra, 2250 or 2270. In addition, background is required in planar curves, velocity and acceleration vectors from Physics 2210 or Math 2210 (Calculus III), or their equivalent courses. The co-requisite is Physics 2210 (Phy 301 before 1998), with actual use of physics minimal. The course uses partial derivatives, the Jacobian matrix and the chain rule in several variables.

To cooperate with the engineering programs on campus, some `maple` contact is required in the course work for 3150. All computer code examples are supplied in `maple` only.

A passive knowledge of `maple` is assumed. Persons without the passive knowledge of `maple` and `unix` may attend one of the *tutorial lectures* on the subject offered during the second week of the

[1] Pronunciation: In the phrase *Gust of Wind* replace *Wind* by *Sun*.

term. The instructor for these tutorials is Angie Gardiner. The dates and times are published at the 3150 web address cited above.

Angie's web page is `www.math.utah.edu/ugrad/tutoring.html`. Her office is MC 155A in building LCB, next to the Math center, phone 585-9478, email `gardiner@math.utah.edu`.

Persons without computer training and no maple experience can survive with a graphing calculator and Microsoft's `Excel` or the MathWork's `matlab`. Free software exists for PC Intel hardware to duplicate most of matlab's functionality.

If you want to use only `matlab`, then be aware that you must translate maple code examples to matlab code by yourself. Maple can do automatic generation of matlab code from maple code. See `CodeGeneration[Maple]` in maple help. Generally, such translations require editing by experienced programmers.

## Course material and requirements

This course is an introduction to Fourier series and partial differential equations in engineering and science. Chapters 1 to 4 and 7 in the Asmar text and class notes published as PDF `www` documents will make up the course material. If you study in isolation, then please know that all topics are enriched in class. Your grade in the course may be reduced by isolation, because the enriched material is tested on exams.

| **Week** | **Section** | **Topic** |
|---|---|---|
| 1 | 1.1-1.2 | Introduction |
| 2 | 2.1-2.2 | Periodic Functions and Fourier Series |
| 3 | 2.3-2.4 | More on Fourier Series |
| 4 | 3.1 | Examples in Physics and Engineering, Midterm 1 |
| 5 | 3.2-3.3 | One Dimensional Wave Equation |
| 6 | 3.4 | D'Alembert's Method |
| 7 | 3.5-3.6 | One Dimensional Heat Equation |
| 8 | | Fall Break |
| 9 | 3.7 | Two Dimensional Equations |
| 10 | 3.8-3.9 | Laplace's and Poisson's Equations |
| 11 | 4.1-4.2 | Circular Coordinate and Vibration |
| 12 | 4.3-4.4 | Laplace's Equation in Circular Regions |
| 13 | 4.4 | Laplace's Equation in Circular Regions, Midterm 2 |
| 14 | 7.1 | Fourier Integral Representation |
| 15 | 7.2-7.3 | Fourier Transform |
| 16 | 7.4-7.5 | Heat Kernel and Poisson Integral Formula |

**Grading**:

Final grades will be based on:

Problems, which are the major part of the **dailies**.

Several computer projects form the minor part of the **dailies**. Each project is counted like several textbook problems.

Two written midterm examinations.

A university-scheduled final exam. This in-class 2-hour examination counts as two additional midterm scores.

**Written In-Class Exams**:

There are two (2) midterm exams. There is a 2-hour in-class final exam as scheduled by the university. The midterm and final exams are graded by G.B. Gustafson.

**Missed exams**. Please send email to `ggustaf@math.utah.edu`. Phone 581-6879 (office) or leave a message with Paula 581-6851. Please know that once you miss the exam, the crisis has ended, and recovery is the next plan. Please respond ASAP.

**Hand-written Dailies**:

There will be less than 60 required dailies due during the semester, including textbook problems and several maple labs.

**Records**:

Accounting of exams and the dailies is initially on paper and ultimately by spreadsheet records. The electronic records are web-posted, without names. Lookup of your electronic record requires that you know your exam scores and a few daily scores. During the course, the first electronic record is printed and distributed in class like returned homework. This usually happens before the final exam. In any case, web-posted electronic records are available after the final exam and for months after the course ends.

## Homework, computer labs, midterms and final

### Textbook problems

The problems are not taken directly from the textbook. They are re-statements of textbook problems, independent of any edition of Asmar's book, distributed as a PDF file at the course web site. The dailies are keyed to Asmar's textbook by section number.

The problems to be submitted for grading are listed at the end of the syllabus, with tentative dues dates. Visit the web site for extra copies of the syllabus.

A duplicate of the tentative due dates appears on the **due dates page** at the course web site. The actual due dates for problems appear only on this web page. Dates are dynamically updated to reflect the reality of what was discussed in class. Ideally, problems are submitted shortly after class discussion, on the planned date, or one day thereafter.

Students are asked to complete each textbook problem and submit their work in their own handwriting. Collaboration is permitted and encouraged on textbook problems in teams of 2 and study groups of a larger size. Submit a separate handwritten report for each partner.

Homework problems are written as one problem per package with your name, class time and a problem label. **Please write 11:50 on your paper, next to your name, and insert a problem label**. Problem labels look like **1.2-5**, for problem 5 in section 1.2 of Asmar. Page numbers depend on the edition of the book – please don't use them.

When several problems are submitted together, then please construct a stapled package with the problems in order. Graders try to write the scores on the top page. This plan speeds up return of the work, so please use stapled packaging when it makes sense. No stapler? Just deposit your work in a stack without the staple.

There are certain **rules** for writing up the textbook problems. A full accounting of the *format suggestions* contributed by students of 2250 appears on the internet course page as a pdf document *format for submitted work*. Please apply the ideas found there to your written work.

### Computer projects

There will be several computer projects assigned during the semester, related to the classroom material. Each project counts the same as 2 to 4 daily problems from the textbook. They will be written by hand and use the software package `maple`.

Maple labs are submitted in a stapled package containing all requested parts.

There is a Math Department Computer Lab in building LCB at which registered students automatically own accounts, and there are other unix labs around campus where `maple` is also available, for example at the College of Engineering CADE lab. Most unix labs can launch remote tty sessions on math hosts using command `ssh user@xserver.math.utah.edu`. To operate in X-windows, replace `ssh` by `ssh -X` or `ssh -X -Y`. Remote files on math hosts can be transferred to your local unix computer with `sftp`. For information on how to do the same for personal computers, visit the campus computer help sites.

## Midterm exam details

The exam will be designed from the sample exam, which is supplied a week before the exam date. A problem on the actual exam is guaranteed to have some spin, a change made to the sample exam problem.

Books, tables, notes and calculators are not allowed on exam day.

An in-class Midterm exam has different presentation rules, and none of the daily problem rules apply in this case. Basically, the in-class exam is a first draft. No answer checks are expected.

## Final exam details

Two hours are reserved for this written exam. As published by the university:

### 11:50 class

The final exam is comprehensive. It covers explicitly chapters 1, 2, 3, 4, 7 with weight distributed evenly across the chapters listed. A study guide consisting of problem types by chapter appear at the web site.

No notes, calculators, tables, books or aids of any kind are allowed on the final exam. Please bring pencils and eraser. Paper will be supplied.

## Due dates and late work

### Due dates

Due dates are updated dynamically at a link on page

`http://www.math.utah.edu/~gustafso/index3150.html`.

Browse this site often. To repeat: **the due dates are not given in class**! Sometimes, email communication about due dates and exams will be made from the registrar's list.

Please prepare submitted work according to the tentative schedule of due dates. The actual due date is the same date, or one day later, as documented at the web site.

### Email notification

You will be sent email about due dates, exam reviews and exam dates during the semester. This service depends on your email address being up to date.

Look up your campus information data by visiting the registrar's campus WWW site (where you add classes). Find out your email address, then test it by emailing a message to yourself. To update the information, return to the registrar's site and edit your personal data.

### When is work late?

Work is considered late and therefore unacceptable when the graded stack has been returned. The state of submitted work is locked at the point when the stack is returned. This record is never appended, it is only corrected for errors.

Are you an exception? It is better to ask than to assume anything.

The lowest five (5) dailies are dropped from consideration in order to eliminate makeups. There is no distinction between a problem from the textbook and a maple lab problem, they earn the same credit.

If more than ten (10) textbook problems have zero scores, then please call 581–6879 or email `ggustaf@math.utah.edu` and discuss the situation and options for completing the work.

### Withdrawal

It is the Math Department policy, and mine as well, to grant any withdrawal request until the University deadline. This promise also means that such a withdrawal requires no explanation. Withdrawals are always initiated by the registered student. All paperwork is the duty of the student. My job is the signature.

### ADA statement

The American with Disabilities Act requires that reasonable accommodations be provided for students with physical, sensory, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss accommodation (113 JWB or 581-6879), which is to say, accommodation shall be made.

## Grading details

### Grading Scale

```
A  = 95-100, A- = 92-94, B+ = 88-91, B  = 84-87,
B- = 80-83,  C+ = 75-79, C  = 65-74, C- = 60-64
```

This scale is determined from 40% passing use GPA increments. It is used for grading and for final letter grade reporting. This scale is for internal use only. Fractional scores are truncated (not rounded) when fitting a score to a letter grade – see below for examples.

### Final grade

The letter grade is determined from the *Grading Scale* above as follows:

$$\text{Final Grade} = \frac{30}{100}(\text{Dailies Average}) + \frac{70}{100}(\text{Midterm+Final Average}).$$

An example: the Dailies Average for 90 textbook problems and maple labs is 91% and the Exam Average of the two midterms and the final exam is 86%. The final grade is $0.3(91) + 0.7(86) = 87.5\%$, which by the scale above is a $B$. While 87.5 rounds to 88, a $B+$, the deciding factor is really the exam average of 86, which is squarely a $B$. The final grade is $B$. If the dailies average was 93 or higher, then the final grade would be 88.1 or higher for a B+.

A precise description of the method of assigning letter grades follows. First, compute the course average $A_v = 0.3A_1 + 0.7A_2$ from the dailies average $A_1$ and the exam average $A_2$. Truncate $A_v$ to an integer (e.g., $A_v = 94.96$ truncates to 94). Assign a letter grade $L$ according to the grading scale (see above). Look at the final exam score $F$ and the exam average $A_2$. If $F$ would give a higher letter grade, then change $L$ to the next possible higher letter grade, e.g., change a $B+$ to an $A-$ (but not $B+$ to $A$). In some cases, when $F$ is low or $A_1$ is low, the average $A_2$ will be used to decide on the letter grade. An example: $A_v = 94.96$, $F = 92$, $A_1 = 94.86$, $A_2 = 95$. The letter grade is $A-$, but the exam average is 95 or $A$, therefore the letter grade $A-$ should be promoted to an $A$.

Promotion based on $A_2$ can change only one level, e.g., $B+$ to $A-$ but not $B+$ to $A$. It is possible with a very low daily average $A_1$ to have rank one or two in the class and yet earn a final grade of **B+**. This happens because the influence of the dailies score is just 30%. An example: final exam grade $F = 100$, exam average $A_2 = 99$, dailies $A_1 = 55$. Then

$$A_v = 0.3 * 55 + 0.7 * 99 = 85.8, \quad L = \mathbf{B} \text{ promoted to } L = \mathbf{B}+.$$

### Rite of passage

A passing grade in the course requires submission of at least 50% of the dailies (includes computer projects). A grade of $E$ is assigned if less than the required 50% is submitted.

The right of passage is absolute, similar to the European system, which requires a body of work to be presented before written and oral final exams are taken. For example, the Czech *vypočet* is a requirement to show a body of completed work as the entrance requirement to administration of written and oral final exams.

### Engineering School Expectations

Mechanical, electrical and chemical engineering departments expect stellar performance in partial differential equations. They have asked for homework to be collected and graded. All of it, not some minimal portion of the total homework.

The various engineering and science departments served by 3150 decide the list of topics to be taught, not the math department.

Graduation requirements of a **C-** or better in required math courses seem to be counter to departmental requests. Be advised that a grade of **C-** is viewed these days as a failure by all graduate schools and employers.

### How dailies are graded

Scores are assigned to dailies as one of 100, 55 or 0. Based on grading history, about 90% of the scores on a given problem are 100%. A score of 0 is routinely given for work not submitted.

**A grade of 100** usually means a complete, correct solution was written. Grading is usually generous, giving a 100 score when the solution method is correct, even if the details contain arithmetic errors and a few missing steps. Flaws in logic are not excused, even if the correct answer was found, due to multiple errors canceling the logic error.

**A grade of 55** means the written work lacked essential details. This score is often given for a written solution with just the answer and a few sketchy details. Examples of sketchy solutions, worth 55 or 0 for a score, appear in the textbook's solution manual. The naive assumption that the textbook's authors have supplied a solution to the problem in the manual is simply false: it is a solution sketch, by design devoid of essential details.

The wrong answer is a reason to look for a flaw. Because all written solutions must have an answer check, a wrong answer signals a careless written report.

## Purpose of the textbook problems

The *purpose* of the problems is to practice doing mathematics, that is, to write out in detail the solutions to problems. A textbook problem is generally an engineering–style "crank" problem, usually devoid of proofs. The process:

- **Understand the problem**. Understanding usually involves reading the textbook. Answers are usually not provided. You may be given an outline of the solution, to increase the probability that the project gets completed on schedule. Problems are discussed in class in finer detail. Slides cover a similar problem or sometimes the exact problem considered in the project. Classroom slides are mirrored at the web site as pdf files, ready to print from an internet browser.

- **Background reading**. To solve a problem, a second opinion of the theory and method is essential. It might be that you can flesh it out of your book's examples, the college algebra text, the calculus text or some engineering mathematics book. No matter, go to a source that works for you. This is *reading* and not a tutorial.

- **Scratch Paper Write–up**. The initial creation of a solution is the essence of the learning process. Everyone learns by repetition, and here is where you do it. Use a pencil and a big eraser, lots of paper, and flesh out a first draft at full speed. Don't submit this draft!
- **Final Copy**. The final copy of the solution uses the scratch paper draft as raw material to *present* the details of the solution. As such, it is more than a collection of formulas on paper. There is no strict requirement, except that *neatness* and *completeness* are a must.
- **Final Copy Format**. The most successful format to date was invented by several engineering mathematics students over the years 1990–2009. This format is described in some detail below and also in the internet document *format for submitted work*.

### Suggestions for Improving Written Work

1. Use engineering paper or plain white paper. Lined notebook paper and graph paper are not acceptable for mathematics, because they introduce flaws in vertical white space.
2. Reports are hand-written in pencil. They start with a problem statement followed by the book's answer or by a final answer summary. Supporting material appears at the end, like a tax return.
3. Every report has an answer check. For problems without an answer, a full answer check is expected.
4. Mathematical notation is on the left, text on the right, about a 60% to 40% ratio. One equal sign per line. Justify equations left or align on the equal signs.
5. Text is left-justified on the right side. It includes explanations, references by keyword or page number, statements and definitions, references to delayed details, like long calculations, graphics and answer checks.
6. Suggestions 4 and 5 can be broken. They are suggestions, not rules.

**Cooperative efforts** are allowed and encouraged. Kindly produce individual handwritten reports. There is no penalty for getting help from others – it is encouraged. This includes tutorial staff in the Math Center (basement of LCB), teaching assistants, faculty and fellow students.

**English language deficiencies** are tolerated but not excused. Short, precise English comments are expected. If English is your second language, then try to improve your writing skills: (1) shorten comments and (2) use page references to the textbook.

**Presentation** is expected to improve throughout the 15 weeks of the course. You are not expected to be an expert in the first week. Correctness of answers will be checked. Problem notes might contain answers plus a solution outline. In class, further details are communicated. Your job is to *improve* on the initial start into the solution. Add the particulars, make comments, and chase down the details from algebra and calculus. College algebra and calculus skills need constant and careful review. Writing up the solution identifies the stumbling blocks and forces a review of background material.

**References** are required on the first occurrence. After that, omit the citation. It is appropriate to refer to the previous assignment on which the citations originated. A statement like *References parallel Exercises 1-5* is enough.

## Tentative Dates for 3150 Submitted work F2009

Aug 31 or Sept 1 in 113 JWB
- Prob1.0-1 (Partial Derivatives)
- Prob1.0-2 (Jacobian)
- Prob1.0-3 (Directional Derivative)

Sept 2
- Prob1.0-4 (First Order ODE)
- Prob1.0-5 (Wronskian)
- Prob1.0-6 (Newton's Laws)

Sept 7
- Prob1.1-3 (PDE Solution)
- Prob1.1-6 (Change of Variables)
- Prob1.1-12 (Method of Characteristics)

Sept 9
- Prob1.2-4a (d'Alembert's Solution)
- Prob1.2-5 (Vibrating Infinite String)

Sept 14
- Prob1.2-17a (Vibrating Finite String)

Sept 16
- Prob2.0-1 (Trigonometric Identities and Integrals)
- Prob2.0-2 (Orthogonality)

Sept 21
- Prob2.1-1 (Periodic Functions)
- Prob2.1-8 (Sums of Periodic Functions)

Sept 23
- Prob2.2-5 (Fourier Series Partial Sum Plots)
- Prob2.2-8 (Fourier Series Computation and Graphics)

Sept 28
- Prob2.0-3 (Even and Odd Functions)
- Prob2.0-4 (Periodic Extensions)

Sept 30
- Prob2.3-7 (Fourier Series Arbitrary Period)
- Prob2.0-5 (Even and Odd Periodic Extensions)

Oct 5
- Prob2.0-6 (Dirichlet Kernel Identity)
- Prob2.4-7 (Half-Range Expansions)
- Prob2.4-15 (Half-Range Sine Expansion)

Oct 7
- Prob2.6-6 (Complex Fourier Series)
- Prob2.6-11 (Series Identities)

Oct 19
- Prob3.1-3 (Classification)
- Prob3.1-7 (Laplace Equation)

Oct 21
- Prob3.2-1 (Wave Equation)
- Prob3.3-9a (Separation of Variables)
- Prob3.3-9b (Snapshot Plots)
- Prob3.3-9c (Surface Plot)

Oct 26
- Prob3.3-13 (Damped Vibrations of a String)
- Prob3.4-15 (d'Alembert's Solution)

Oct 28
- Prob3.5-13 (Nonhomogeneous Heat Equation)
- Prob3.6-3 (Heat Conduction in an Insulated Bar)

Nov 2
- Prob3.7-5a (Vibrations of a Membrane)
- Prob3.7-5b (Membrane Snapshots)

Nov 4
- Prob3.7-12 (Heat Conduction in a Plate)
- Prob3.8-2 (Steady-State Temperature in a Plate)

Nov 9
- Prob3.9-3 (Poisson Problem)

Prob4.1-5 (Laplacian in Spherical Coordinates)
Prob4.0-1 (Power Series Method)
Prob4.0-2 (Euler Differential Equation)

Nov 11

Prob4.0-3 (Frobenius Method)
Prob4.0-4 (Frobenius Method Case 3)

Nov 16

Prob4.2-1 (Radially Symmetric Drumhead)
Prob4.3-3 (Non-Symmetric

Nov 18

Prob4.4-5 (Dirichlet Problem on a Disk)
Prob4.4-15a (Exterior Dirichlet Problem on a Disk)
Prob4.4-15b (Cartesian Coordinates)

Nov 23

Prob4.4-15c (Isotherms)
Prob7.1-8a (Fourier Integral Formulas)
Prob7.1-8b (Fourier Integral Convergence)

Nov 25

Prob7.1-19 (Fourier Integral and Integration Formulas)
Prob7.2-20 (Fourier Transform)

Nov 30

Prob7.2-18a (Fourier Transform Rules)
Prob7.2-18b (Fourier Transform Table)

Dec 2

Prob7.2-31a (Fourier Transform Calculus)
Prob7.2-45 (Fourier Transform Convolution)

Dec 7

Prob7.3-1 (Fourier Transform Method Wave Equation)
Prob7.3-4 (Fourier Transform Method Heat Equation)

Chapters 1,2,3. Last possible date: Nov 25, under the door 113 JWB

Xc1.2-4b (d'Alembert's Solution)
Xc1.2-17b (Loudness)
Xc2.1-8 (Periodic Functions)
Xc2.1-18 (Change of Variables)
Xc2.1-20 (Floor Function)
Xc2.2-15 (Fourier Series Computation)
Xc2.3-32 (Failure of Term-by-Term Differentiation)
Xc2.3-34 (Term-by-Term Integration)
Xc3.4-18 (Energy Conservation and d'Alembert's Solution)
Xc3.7-12 (Heat Conduction in a Plate)

Chapters 4,7. Last possible date: Dec 15, under the door 113 JWB

Xc4.1-9 (Spherical Laplacian Symmetric Case)
Xc4.2-12a (Series Identity for J_0(x))
Xc4.2-12b (Bessel Function Identities)
Xc4.3-13 (Two-Dimensional Heat Conduction)
Xc4.4-11 (Dirichlet Series Formula)
Xc7.2-31a (Fourier Transform Calculus)
Xc7.2-47 (Fourier Transform Convolution)
Xc7.3-17 (Fourier Transform Method Infinite Beam)
Xc7.4-2 (Heat Kernel)
Xc7.4-6 (Heat Kernel)

**Policy on Dailies**: The lowest 4 of the 66 dailies will be dropped (four problems, not four collections). Any record with less than 24 daily and lab scores earns a grade of **E**, regardless of midterm and final exam scores [Ch1+Ch2 is more than 24 scores]. Deadlines set at web site `www.math.utah.edu/~gustafso/`. Work not in the stack earns a grade of zero.

**Policy on Exams**: The final exam is doubled before determining the exam average, to count like two midterms.

**Grading Scale**: $A$ = 95-100, $A-$ = 92-94, $B+$ = 88-91, $B$ = 84-87, $B-$ = 80-83, $C+$ = 75-79, $C$ = 65-74, $C-$ = 60-64.