SAMPLE Solution

2.4-3. Apply Euler's method twice to evaluate $y(1/2)$, first with $h = 0.25$ and second with $h = 0.1$. Compare the approximations with the exact value.

$$\begin{cases} y' = y + 1 \\ y(0) = 1 \end{cases}$$

Exact solution $\quad y(x) = 2e^x - 1$

Derivation:

$y' - y = 1$	Standard linear form
$\dfrac{(e^{-x}y)'}{e^{-x}} = 1$	Replace LHS by integrating factor fraction $(Qy)'/Q$; $Q = e^{\int(-1)dx} = e^{-x}$.
$(e^{-x}y)' = e^{-x}$	Quadrature preparation.
$e^{-x}y = -e^{-x} + c$	Apply quadrature
$y = -1 + ce^{x}$	General solution
$y = -1 + 2e^{x}$	Evaluate $c = 2$ from $y(0) = 1$.

Ans Check: problem 2.4-3 p 119 E&P.

Comparison Table

$y(1/2), h = 0.25$	$y(1/2), h = 0.1$	$y(1/2)$, Exact
2.1250	2.22102	2.297442542

Data extracted from Dot Tables in maple worksheet appendix.
Answers match the textbook answers.

Hand solution steps 2.4-3

$\boxed{h=0.25}$ $y' = y+1,\ y(0)=1$

$f(x,y) = y+1$ — RHS of $y' = f(x,y)$

$x_0 = 0,\ y_0 = 1$ — From $y(0)=1$

$y = y_0 + h\, f(x_0,y_0)$ — Euler algorithm

$= 1 + 0.25\,(y_0+1)$

$= 1.5$

Dots[2] = [0.25, 1.5] — Line 2 of Dots Table, $h=0.25$

Ans check: matches Dots Table in maple worksheet appendix.

$\boxed{h=0.1}$ $y' = y+1,\ y(0)=1$

$x_0 = 0,\ y_0 = 1,\ h = 0.1$ — From $y(0)=1$

$y = y_0 + h\, f(x_0,y_0)$

$= y_0 + h\,(y_0+1)$

$= 1 + 2h$

$= 1.2$

Dots[1] = [0.1, 1.2] — Line 2 of Dots table for $h=0.1$

Graphics 2.4-3

See The maple worksheet appendix

Maple Worksheet Appendix

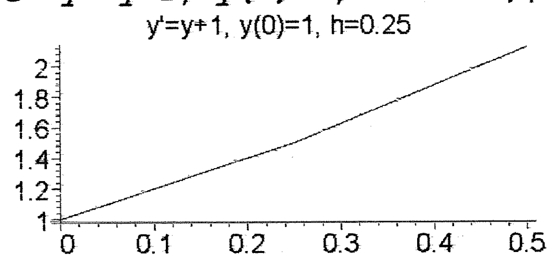
2.4-3 Edwards and Penney

```
> # 2.4-3(h=0.25) Euler. Group 1, initialize.
  f:=unapply(y+1,(x,y)):
  x0:=0: y0:=1: h:=0.25: Dots1:=[x0,y0]:
> # Group 2, repeat 2 times. Euler's method
   Y:=y0+h*f(x0,y0);
   x0:=x0+h:y0:=Y:Dots1:=Dots1,[x0,y0];
```

$$Y := 2.1250$$

$$Dots1 := [0, 1], [.25, 1.50], [.50, 2.1250]$$

```
> # Group 3, plot.
   plot([Dots1],title="y'=y+1, y(0)=1, h=0.25");
```



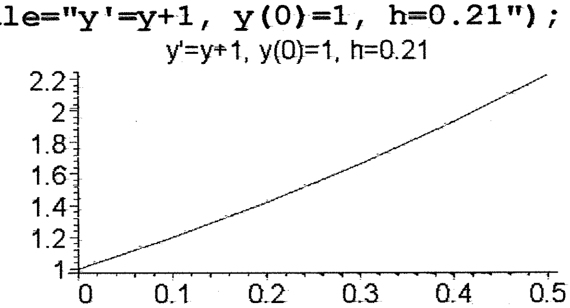

```
> # 2.4-3(h=0.1) Euler. Group 1, initialize.
  f:=unapply(y+1,(x,y)):
  x0:=0: y0:=1: h:=0.1: Dots2:=[x0,y0]:
> # Group 2, repeat 5 times. Euler's method
   Y:=y0+h*f(x0,y0);
   x0:=x0+h: y0:=Y: Dots2:=Dots2,[x0,y0];
```

$Y := 2.22102$

$$Y := 2.22102$$

$$Dots2 := [0, 1], [.1, 1.2], [.2, 1.42], [.3, 1.662], [.4, 1.9282], [.5, 2.22102]$$

```
> # Group 3, plot.
   plot([Dots2],title="y'=y+1, y(0)=1, h=0.21");
```




```
> # Exact solution at x=1/2
  exacty:=unapply(2*exp(x)-1,x): evalf(exacty(1/2));
```

$$2.297442542$$

```
> plot(exacty(x),x=0..5);    # plot matches one above - not printed here to save
                             # paper and pdf file size= GB6
```