Name. _____

Applied Differential Equations 2250 Sample Midterm Exam 1 Tuesday, 9 February 2006 Wednesday, 10 February 2006 arranged

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. (Quadrature Equation)

Solve for the general solution y(x) in the equation $y' = xe^{-x} + \csc^2 x + 2\sec^2 x + 16x^4$

 $\overline{1+4x^2}$

[Integral tables will be supplied for anything other than basic formulae. This sample problem would require no integral table. The exam problem will be shorter.]

2. (Separable Equation Test) The problem

$$y' = x \left(e^{2x} x^{2/3} y e^y + \sin(x) y e^y \right) - x \sin(x) \sin(y) - e^{2x} x^{5/3} \sin(y)$$

may or may not be separable. If it is, then write formulae for F, G which decompose the problem as y' = F(x)G(y). Otherwise, explain in detail why it fails to be separable. Do not solve for y!

3. (Solve a Separable Equation) Given $yy' = \frac{6x^3 + 12x}{1+x}(1-4y^2)$, (a) Find all equilibrium solutions,

- (b) Find the non-equilibrium solution in implicit form.

Do not solve for *y* explicitly.

- (a) Solve $2v'(t) = -64 + \frac{4}{t+1}v(t)$, v(0) = 3. Show all integrating factor steps. (b) Using the answer v(t) from (a), solve y'(t) = v(t), y(0) = 2. Show all quadrature steps.

5. (Stability)

(a) Draw a phase line diagram for the differential equation

$$dx/dt = (2 - 5x)^3 (1 - 2x)(1 - 4x^2).$$

Expected in the diagram are equilibrium points and signs of x' (or flow direction markers \langle and \rangle).

(b) Draw a phase diagram using the phase line diagram of (a). Add these labels as appropriate: funnel, spout, node, stable, unstable. Show at least 10 threaded curves. A direction field is not required.