Name. $\qquad$

## Applied Differential Equations 2250 Sample Midterm Exam 1 <br> Tuesday, 9 February 2006 Wednesday, 10 February 2006 arranged

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count $75 \%$. The answer counts $25 \%$.

## 1. (Quadrature Equation)

Solve for the general solution $y(x)$ in the equation $y^{\prime}=x e^{-x}+\csc ^{2} x+2 \sec ^{2} x+$ $\frac{16 x^{4}}{1+4 x^{2}}$.
[Integral tables will be supplied for anything other than basic formulae. This sample problem would require no integral table. The exam problem will be shorter.]

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## 2. (Separable Equation Test)

The problem

$$
y^{\prime}=x\left(e^{2 x} x^{2 / 3} y e^{y}+\sin (x) y e^{y}\right)-x \sin (x) \sin (y)-e^{2 x} x^{5 / 3} \sin (y)
$$

may or may not be separable. If it is, then write formulae for $F, G$ which decompose the problem as $y^{\prime}=F(x) G(y)$. Otherwise, explain in detail why it fails to be separable. Do not solve for $y$ !

Use this page to start your solution. Attach extra pages as needed, then staple.

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3. (Solve a Separable Equation)

Given $y y^{\prime}=\frac{6 x^{3}+12 x}{1+x}\left(1-4 y^{2}\right)$,
(a) Find all equilibrium solutions,
(b) Find the non-equilibrium solution in implicit form.

Do not solve for $y$ explicitly.

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## 4. (Linear Equations)

(a) Solve $2 v^{\prime}(t)=-64+\frac{4}{t+1} v(t), v(0)=3$. Show all integrating factor steps.
(b) Using the answer $v(t)$ from (a), solve $y^{\prime}(t)=v(t), y(0)=2$. Show all quadrature steps.

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## 5. (Stability)

(a) Draw a phase line diagram for the differential equation

$$
d x / d t=(2-5 x)^{3}(1-2 x)\left(1-4 x^{2}\right) .
$$

Expected in the diagram are equilibrium points and signs of $x^{\prime}$ (or flow direction markers $<$ and $>$ ).
(b) Draw a phase diagram using the phase line diagram of (a). Add these labels as appropriate: funnel, spout, node, stable, unstable. Show at least 10 threaded curves. A direction field is not required.

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