Introduction to Linear Algebra 2270-1
Sample Midterm Exam 3 Fall 2003
In-class Exam Date: Wednesday, November 19, 2003

Instructions. Calculators are not allowed. Books and notes are not allowed. Time: 15 minutes. You will be given a variety of problems from which to select. The object is to solve two of them in 15 minutes. The longer ones will be identified as worth two problems.

5. **(Exam 3 in-class)**

(a) Prove $\mathbf{v}_1$, $\mathbf{v}_2$ orthogonal and A an $n \times n$ orthogonal matrix implies $A\mathbf{v}_1$, $A\mathbf{v}_2$ orthogonal.

(b) Prove $\left(\sum_{k=1}^{n} x_k\right)^2 \le n \sum_{k=1}^{n} |x_k|^2$.

(c) Prove or disprove: A orthogonal implies A^2 orthogonal.

(d) Let A be $n \times n$ with eigenpairs $(\lambda_i, \mathbf{v}_i)$, $1 \le i \le n$. Prove that $(A - \lambda_1 I) \cdots (A - \lambda_n I) = 0$.

(e) Let V be a subspace of $\mathcal{R}^n$. Prove that V and $V^{\perp}$ meet only in the zero vector.

(f) Let A be $m \times n$. Prove that $\mathbf{ker}(A) = \{\mathbf{0}\}$ implies $A^T A$ is invertible.

(g) Let V be an inner product space. Suppose that $\mathbf{v} = \sum_{k=1}^{n} c_k \mathbf{v}_k$ and $\{\mathbf{v}_k\}$ is an orthogonal set. Compute c_2.

(h) Find a 3×3 matrix A such that $\det(A - \lambda I) = -\lambda^3 + 15\lambda^2 - 3\lambda + e^2$.