Name. ________________________ **Section**. __________

Applied Differential Equations 2250-1 and 2250-3
Midterm Exam 4, Due classtime 27-Nov-2002

Instructions. The four take-home problems below are to be submitted by Wednesday, November 27. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.

The in-class portion of the exam (December 2) is 15 minutes, one problem, of a type similar to one of the last two problems. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

1. **(Eigenanalysis)** Find the 3×3 matrix P which under the change of variables $x = PY$ converts the system $x' = Ax$ into $Y' = DY$, where

$$A = \begin{pmatrix} -1 & -7 & -3 \\ 0 & 3 & 0 \\ 0 & -1 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Represent the general solution of $x' = Ax$ as a matrix product, by solving $Y' = DY$ and then back-substituting the answer into the relation $x = PY$.

2. **(Coupled spring-mass system)** The system

$$\begin{aligned} x_1'' &= -k_1x_1 + k_2(x_2 - x_1), \\ x_2'' &= -k_2(x_2 - x_1) + k_3(x_3 - x_2), \\ x_3'' &= -k_3(x_3 - x_2) - k_4x_3 \end{aligned} \qquad (1)$$

represents three masses m_1, m_2, m_3 coupled by springs of Hooke's constant k_1, k_2, k_3, k_4 as in Figure 7.4.1, Edwards-Penney. Let $m_1 = m_2 = m_3 = 1$, $k_1 = k_2 = k_3 = k_4 = 1$. Find the natural frequencies ω_1, ω_2, ω_3 of ocillation of system (1). *Do Not Solve* for x_1, x_2, x_3!

3. **(Laplace transform)** Solve $x'' + x = \sin 2t$, $x(0) = 0$, $x'(0) = 0$ by two methods: (1) Undetermined coefficients and (2) Laplace transform. Show all steps, thus verifying the answer $x = (2\sin t - \sin 2t)/3$.

4. **(Laplace inverse transform)** Show the partial fraction steps involved in solving for $f(t)$ in the Laplace equation

$$\mathcal{L}(f(t)) = \frac{2s}{(s-1)(s-2)(s^2+1)}.$$

Kindly flag the step where Lerch's theorem is applied to give the answer $f(t) = -e^t + \frac{4}{5}e^{2t} + \frac{1}{5}\cos(t) - \frac{3}{5}\sin(t)$.