Name. _____ **Section**. _____

Applied Differential Equations 2250-1 and 2250-3
Midterm Exam 3, Due classtime 13-Nov-2002

Instructions. The four take-home problems below are to be attached to your in-class exam. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.

The in-class portion of the exam is 15 minutes, one problem, of a type similar to one of the four problems. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

1. **(Variation of Parameters)** Show the steps in the solution of

$$y'' - y = xe^x$$

by variation of parameters, to obtain the general solution

$$y = c_1 e^x + c_2 e^{-x} + \frac{1}{4}(x^2 - x)e^x.$$

Use formula (3), page 335.

2. **(Undetermined Coefficients)** Show the steps in the solution of

$$y''' - y' = x + xe^x - \sin x$$

by undetermined coefficients, to obtain the general solution

$$y = c_1 e^x + c_2 e^{-x} + c_3 + \frac{1}{4}(x^2 - 3x + 7/2)e^x - \frac{1}{2}x^2 - \frac{1}{2}\cos x.$$

3. **(Practical Resonance)** Given $x'' + 10x' + 650x = 100\cos(\omega t)$, find
   (a) the steady-state solution $x = A\cos(\omega t) + B\sin(\omega t)$,
   (b) the amplitude $C(\omega)$,
   (c) the practical resonant frequency ω^*.
   Use formulas on pages 346–347. Show all steps used to obtain the answers.

4. **(RLC Circuit)** Find an RLC-circuit equation $LQ'' + RQ' + (1/C)Q = E(t)$ whose general solution is given by the charge equation

$$Q(t) = c_1 e^{-2t}\cos(\sqrt{3}t) + c_2 e^{-2t}\sin(\sqrt{3}t) + \sin(t) - 5\cos(t).$$

Hint: Apply superposition to find Q_h and Q_p. Once you choose L, R, C, then the differential equation when $Q = Q_p$ determines $E(t)$.