Name. _____ **Section**. _____

Applied Differential Equations 2250-1 and 2250-3
Midterm Exam 2, Due classtime 18-Oct-2002

Instructions. The four take-home problems below are to be attached to your in-class exam. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.

The in-class portion of the exam is 15 minutes, one problem, of a type similar to either problem 3 or problem 4. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

1. **(Periodic harvesting)** The population equation $y' = y(1-y) - \sin(5.57t)$ appears to have a steady-state periodic solution that oscillates about $y = 1$. (a) Apply ideas from the example below to make a computer graphic that supports this conclusion. (b) Discuss the biological meaning.

```
with(DEtools):  # See Figure 12, section 2.5
de:=diff(y(t),t)=y(t)*(2-y(t))-4*cos(4*Pi*t):
ic:=[y(0)=1.7],[y(0)=2],[y(0)=2.4],[y(0)=2.8]:
DEplot(de,y(t),t=0..4,y=1..3,[ic],stepsize=0.05);
```

2. **(Cross bow)** The height $y(t)$ of a crossbow bolt shot straight upward satisfies $v'(t) = -(0.0013)v(t)|v(t)| - 9.8$, $v(0) = 50$, $y(0) = 0$, where $v = dy/dt$, in `mks` units. Compute decimal approximations for the maximum height, the ascent time, the fall time and the impact speed. See Example 3, section 2.3.

3. **(Gaussian algorithm)** Solve for x, y, z in the 3×3 linear system

$$\begin{array}{rcrcrcl} 2x & + & 3ay & + & cz & = & b \\ 3x & + & ay & + & 2cz & = & 0 \\ 5x & + & 4ay & + & 3cz & = & b \end{array}$$

using the Gaussian algorithm, for all constant values of a, b, c. Include all algorithm details and an answer check (there are four cases and four separate answers).

4. **(Inverse matrix)** Determine by `rref` methods the inverse matrix of

$$A = \begin{pmatrix} 1 & a & 0 \\ 3 & 0 & b \\ 0 & 4 & 3 \end{pmatrix}.$$

Please state conditions on a, b for when the inverse exists. Include an answer check.