Name _____   **Class time** _____

Applied Differential Equations 2250-1 and 2250-2
Midterm Exam 3 Spring 2004
Take-Home Problem 1 due at class time 12 Apr

Instructions. The two take-home problems below have separate due dates.

The in-class portion of the exam is 50 minutes. A sample exam appears below as problems 3,4,5. Calculators, study aids and books are not allowed.

1. **(Variation of Parameters)** Show the steps in the solution by variation of parameters, formula (33) page 335, to obtain the given general solution y.

Version A-D:
$$y'' - 4y = 1 - xe^{-2x},$$
$$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{4}$$
$$+ \frac{1}{16} x e^{-2x} + \frac{1}{8} x^2 e^{-2x}.$$

Version E-K:
$$y'' - 16y = x - xe^{-4x},$$
$$y = c_1 e^{4x} + c_2 e^{-4x} - \frac{1}{16} x$$
$$+ \frac{1}{16} x^2 e^{-4x} + \frac{1}{64} x e^{-4x}.$$

Version L-Q:
$$y'' - 4y' = 1 - xe^{4x},$$
$$y = c_1 e^{4x} + c_2 - \frac{1}{4} x$$
$$- \frac{1}{8} x^2 e^{4x} + \frac{1}{16} x e^{4x}.$$

Version R-Z:
$$y'' + 4y' = x - xe^{-4x},$$
$$y = c_1 e^{-4x} + c_2 - \frac{1}{16} x$$
$$+ \frac{1}{8} x^2 + \frac{1}{8} x^2 e^{-4x} + \frac{1}{16} x e^{-4x}.$$

Staple this page to the front of your submitted problem #1.
The version is to match the first letter of your last name.

Name _____   **Class time** _____

Applied Differential Equations 2250-1 and 2250-2
Midterm Exam 3 Spring 2004
Take-Home Problem 2 due at class time 14 April

2. (Undetermined Coefficients) Show the steps in the solution of the differential equation by undetermined coefficients, to obtain the given general solution y.

Version A-D:

$y''' - y'' = x^3 + e^x - \cos(3x)$,

$y = c_1 e^x + c_2 x + c_3 - \frac{1}{20}x^5 - \frac{1}{4}x^4$
$\quad - x^3 - 3x^2 + (x-2)e^x$
$\quad - \frac{1}{90}\cos(3x) + \frac{1}{30}\sin(3x)$.

Version E-K:

$y''' - 4y'' = x + x^3 + e^{4x} - \cos(2x)$,

$y = c_1 e^{4x} + c_2 x + c_3 + (\frac{1}{16}x - \frac{1}{32})e^{4x}$
$\quad - \frac{1}{80}x^5 - \frac{1}{64}x^4 - \frac{11}{192}x^3 - \frac{11}{256}x^2$
$\quad - \frac{1}{20}\cos(2x) + \frac{1}{40}\sin(2x)$.

Version L-Q:

$y''' - y'' = 1 + x^3 + xe^x - \sin x$,

$y = c_1 e^x + c_2 x + c_3 + (\frac{1}{2}x^2 - 2x + 3)e^x$
$\quad - \frac{1}{20}x^5 - \frac{1}{4}x^4 - x^3 - \frac{7}{2}x^2$
$\quad - \frac{1}{2}\cos(x) - \frac{1}{2}\sin(x)$.

Version R-Z:

$y''' + 4y'' = x^3 + x^2 + xe^{4x} - \sin x$,

$y = c_1 e^{-4x} + c_2 x + c_3 + \frac{1}{128}xe^{4x}$
$\quad - \frac{5}{1024}e^{4x} + \frac{1}{80}x^5 + \frac{1}{192}x^4 - \frac{1}{192}x^3$
$\quad + \frac{1}{256}x^2 - \frac{1}{17}\cos(x) + \frac{4}{17}\sin(x)$.

Staple this page to the front of your submitted problem #2.
The version is to match the first letter of your last name.

Applied Differential Equations 2250-1 and 2250-2
Sample In-class Midterm Exam 3 Spring 2004
Exam Date: 16 April

3. (Practical Resonance) Given $x'' + 10x' + 650x = 100\cos(\omega t)$, find:
   (a) The steady-state solution $x = A\cos(\omega t) + B\sin(\omega t)$.
   (b) The practical resonant frequency ω^*.

4. (RLC Circuit) Assume $L = 5$ and the RLC-circuit equation $LQ'' + RQ' + (1/C)Q = E(t)$ has general solution

$$Q(t) = c_1 e^{-t}\cos(\sqrt{2}t) + c_2 e^{-t}\sin(\sqrt{2}t) + \sin(t) - \cos(t).$$

(a) Find the characteristic equation and its roots.
(b) Find $E(t)$.

5. (Particular Solution)
Solve for a particular solution $y_p(x)$. Cite the method(s) used and show all steps.

$$y''' - y' = 2e^{1+\pi} + e^{x-\pi}.$$