Name. ________________________ **Section**. __________

Applied Differential Equations 2250-1 and 2250-2
Midterm Exam 3, Version E-K

Problems 1,2 Due class time 10 Nov 2003

(Problems 3,4 Due class time 14 Nov 2003)

Instructions. Each student must submit their own handwritten report (no joint reports). Answer checks using `maple` assist must attach the `maple` output. Include answer checks where requested.

The 15-minute in-class portion of the exam on 17 November is one problem, of a type similar to one of the four problems. Calculators, notes and books are NOT ALLOWED.

1. **(Variation of Parameters)** Use formula (3), page 335, for variation of parameters.
(a) Compute the Wronskian of $e^{2x}\cos(\ln x^2)$, $e^{2x}\sin(\ln x^2)$ at $x = 1$.
(b) Suppose $y = x^2 + 3 + e^{-x} + (1+x)\sin 2x - \cos^2 x + \sin^2 x$ satisfies $y'' + 4y = F(x)$. Find a particular solution y_p with fewest terms.
(c) Display an integral formula, unevaluated, for the solution to the problem $9y'' + y = \ln|1 + x^2|$, $y(1) = 0$, $y'(1) = 0$.
(d) A Is there a solution $x(t)$ with $\lim_{t\to\infty} x(t) = \infty$ for $x'' + 5x' + 6x = t^2/(1 + t^2)$? Give a proof or counterexample.

2. **(Undetermined Coefficients)** A function $g(x)$ is called an **atom** provided it has one of the forms (where $b > 0$, $k \ne 0$)

(1) $g(x) =$ polynomial	(3) $g(x) =$ (polynomial)$e^{ax}\cos(bx)$
(2) $g(x) =$ (polynomial)e^{kx}	(4) $g(x) =$ (polynomial)$e^{ax}\sin(bx)$

(a) Let $f(x) = -xe^{2x}\cos x + 4\sin x + x^2 + 2x - e^{2x}\cos x$. Decompose $f(x)$ into the fewest number of atoms and classify each atom as type (1), (2), (3), (4).
(b) Suppose the characteristic equation is $r^4 - 9r^2 = 0$ and $y_1 = d_1 e^{-3x}$, $y_2 = d_2 + d_3 x$, $y_3 = d_4\cos 3x + d_5\sin 3x$ are initial trial solutions for certain atoms. Find the roots for the atoms and report the revised trial solutions.
(c) Find a trial solution for $y''' + 9y' = x^2(1 + \cos 3x) - x$. Do not solve for y_p.
(d) Determine the undetermined coefficients in the trial solution $y = d_1 x\cos 2x + d_2 x\sin 2x$ for $y''' + 4y' = 10\cos 2x$. Show all steps. Answer check expected.

Please staple this exam to your solutions and submit it at class time on the due date.

Name. ________________________ **Section.** __________

Applied Differential Equations 2250-1 and 2250-2
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Problems 3,4 Due class time 14 Nov 2003

Instructions. Each student must submit their own handwritten report (no joint reports). Answer checks using `maple` assist must attach the `maple` output. Include answer checks where requested.

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3. (Practical Resonance) Given $x'' + 4x' + 68x = 12\cos(\omega t)$,
(a) Find the steady-state solution $x = A\cos(\omega t) + B\sin(\omega t)$.
(b) Plot the amplitude function $C(\omega)$.
(c) Find the practical resonant frequency ω^*.
(d) Solve for $x(t)$ when $\omega = \omega^* + 0.1$. Graph the steady-state oscillation. `Maple` expected. Answer check expected.
Use formulas on pages 346–347 (don't re-derive book formulas). Show all steps used to obtain the answers.

4. (RLC circuit)
(a) An RLC circuit equation $LQ'' + RQ' + (1/C)Q = E(t)$ has general solution $Q = Q_h + Q_p$ where

$$Q_h(t) = c_1 e^{-3t}\cos(\sqrt{5}t) + c_2 e^{-3t}\sin(\sqrt{5}t),$$
$$Q_p(t) = \cos 4t - 5\sin 4t.$$

Given $R = 5$, find L, C, E. An answer check is expected.

(b) Use L, C from (a). Solve $LQ'' + (1/C)Q = \cos(5t)$, $Q(0) = Q'(0) = 0$. Report the values A, α, β in the **beats formula** $x(t) = A\sin\alpha t \sin\beta t$. Graph $Q(t)$. Book references required (section 5.6).

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