Name. _____ **Section**. _____

Applied Differential Equations 2250-1 and 2250-2
Midterm Exam 2 Spring 2003
Distributed Feb 19, Due March 3

Instructions. The four take-home problems below are to be submitted at class time on March 3. Answer checks are expected. If `maple` assist is used, then please attach the `maple` output.

The February 28 in-class portion of the exam is 15 minutes, one problem, of a type similar to either problem 3 or problem 4. Calculators, hand-written or computer-generated notes are allowed, including xerox copies of tables or classroom xerox notes. Books are not allowed.

1. **(Periodic harvesting)** The population equation $y' = y(1-y) - \sin(2\pi t)$ appears to have a steady-state periodic solution that oscillates about $y = 1$. (a) Apply ideas from the example below to make a computer graphic with 6 solution curves that oscillate about $y = 1$. (b) Find experimentally a threshold population size y_1 so that $y(0) < y_1$ implies $y(t) = 0$ (population dies out) for some later time t, while $y(0) > y_1$ implies $y(t) > 0$ forever and the solution $y(t)$ oscillates about $y = 1$. See Figure 2.5.12, page 128.

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# Example.  See Figure 12, section 2.5
with(DEtools):
de:=diff(y(t),t)=y(t)*(2-y(t))-4*cos(4*Pi*t):
ic:=[y(0)=1.7],[y(0)=2],[y(0)=2.4],[y(0)=2.8]:
DEplot(de,y(t),t=0..4,y=1..3,[ic],stepsize=0.05);
```

2. **(Cross bow)** The height $y(t)$ of a crossbow bolt shot straight upward satisfies $v'(t) = -(0.00105)v(t)|v(t)| - 9.8$, $v(0) = 48$, $y(0) = 0$, where $v = dy/dt$, in `mks` units. Compute decimal approximations for the maximum height and the impact speed. Use freely formulas (12) to (18), pages 98-99. Check your answer by comparison to Example 3 page 100. Reference results found on pp 98-100 by equation number and page (don't derive!).

3. **(Gaussian algorithm)** Solve for x, y, z in the 3×3 linear system

$$\begin{array}{rcrcrcr} 2x & + & 2ay & + & cz & = & b \\ 3x & + & ay & + & 2cz & = & 2b \\ 5x & + & 3ay & + & 3cz & = & 3b \end{array}$$

   using the Gaussian algorithm, for all constant values of a, b, c. Include all algorithm details and an answer check for each of the three separate cases. Sanity check: $a \neq 0$ is one case, with parametric solution $x = 3b/4 - 3ct_1/4$, $y = -b/(4a) + ct_1/(4a)$, $z = t_1$. The case $a = 0$ has subcases $c \neq 0$ and $c = 0$, for one of which you will report *no solution*.

4. **(Inverse matrix)** Determine by `rref` methods the inverse matrix of

$$A = \begin{pmatrix} 1 & a & 0 \\ a & 0 & b \\ 0 & 1 & 1 \end{pmatrix}.$$

   Please state conditions on a, b for when the inverse exists. Show all hand details. Include an answer check, preferably done in `maple`.