Name. _____

Applied Differential Equations 2250
Sample Midterm Exam 1
Review Tuesday, 7 February 2006
Review Wednesday, 8 February 2006 arranged

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. **(Quadrature Equation)**

   Solve for the general solution $y(x)$ in the equation $y' = xe^{-x} + \csc^2 x + 2\sec^2 x + \dfrac{16x^4}{1 + 4x^2}$.

   [Integral tables will be supplied for anything other than basic formulae. This sample problem would require no integral table. The exam problem will be shorter.]

2. **(Separable Equation Test)**
   The problem

   $$y' = x\left(e^{2x}x^{2/3}ye^y + \sin(x)ye^y\right) - x\sin(x)\sin(y) - e^{2x}x^{5/3}\sin(y)$$

   may or may not be separable. If it is, then write formulae for F, G which decompose the problem as $y' = F(x)G(y)$. Otherwise, explain in detail why it fails to be separable. Do not solve for y!

3. **(Solve a Separable Equation)**
   Given $yy' = \dfrac{6x^3 + 12x}{1 + x}(1 - 4y^2),$
   (a) Find all equilibrium solutions,
   (b) Find the non-equilibrium solution in implicit form.
   Do not solve for y explicitly.

4. **(Linear Equations)**
   (a) Solve $2v'(t) = -64 + \dfrac{4}{t + 1}v(t)$, $v(0) = 3$. Show all integrating factor steps.
   (b) Using the answer $v(t)$ from (a), solve $y'(t) = v(t)$, $y(0) = 2$. Show all quadrature steps.

5. **(Stability)**
   (a) Draw a phase line diagram for the differential equation

   $$dx/dt = (2 - 5x)^3(1 - 2x)(1 - 4x^2).$$

   Expected in the diagram are equilibrium points and signs of x' (or flow direction markers $<$ and $>$).
   (b) Draw a phase diagram using the phase line diagram of (a). Add these labels as appropriate: funnel, spout, node, stable, unstable. Show at least 10 threaded curves. A direction field is not required.