Name _____ Class Time _____

Math 2250 Maple Project 3: Numerical Methods
August 2006

Due date: See the internet due dates. Maple lab 3 has six problems L3.1, L3.2, L3.3, L3.4, L3.5, L3.6.

References: Code in `maple` appears in `2250mapleL3-F2006.txt` at URL `http://www.math.utah.edu/~gustafso/`. This document: `2250mapleL3-F2006.pdf`. Other related and required documents are available at the web site:

- Report details on 2.4,2.5,2.6 prob 6
- Report details on 2.4,2.5,2.6 prob 12
- Numerical Solution of First Order DE (typeset, 19 pages, 220k pdf)
- Sample Report for 2.4-3 (pdf 3 pages, 350k)
- Numerical DE coding hints, TEXT Document (1 pages, 2k)
- Sample maple code for Euler, Heun, RK4 (maple worksheet)
- Sample maple code for exact/error reporting (maple worksheet)

Problem L3.1. **(E & P Exercise 2.4-6)**

The actual solution of $y' = -2xy$, $y(0) = 2$ is $y = 2e^{-x^2}$. Apply Euler's method to produce two dot tables, as follows. The first has three row, $h = 0.25$. The second has six rows, $h = 0.1$. Reproduce the table below and fill in the missing values. Follow the sample report for Exercise 2.4-3

`http://www.math.utah.edu/~gustafso/2250SampleProblem2.4-3.pdf`

but exclude derivation and answer check of the actual symbolic solution. Hand written work includes a check of row 2 of the dot table ($x = 0.1$ or $x = 0.25$ only).

h	actual $y(.5)$	approx $y(.5)$
0.25	1.558	1.750
0.1	1.558	

Problem L3.2. **(E & P Exercise 2.5-6)**

The actual solution of $y' = -2xy$, $y(0) = 2$ is $y = 2e^{-x^2}$. Apply Heun's method (Improved Euler) to produce one dot table of six rows, $h = 0.1$. Reproduce the table below. Follow the sample report for Exercise 2.4-3 as in problem L3.1 above. Hand written work includes a check of row 2 of the dot table ($x = 0.1$ only).

x	0.0	0.1	0.2	0.3	0.4	0.5
actual $y(x)$	2.000000000	1.980099667	1.921578878	1.827862371	1.704287578	1.557601566
approx $y(x)$	2.000	1.9800	1.9214	1.8276	1.7041	1.5575

Problem L3.3. **(E & P Exercise 2.6-6)**

The actual solution of $y' = -2xy$, $y(0) = 2$ is $y = 2e^{-x^2}$. Apply the RK4 method to produce one dot table of three rows, $h = 0.25$. Reproduce the table below. Follow the sample report for Exercise 2.4-3 as in problem L3.1 above. Exclude the hand check of row 2 of the dot table, because the computer work is more reliable.

x	0.00	0.25	0.50
actual $y(x)$	2.000000000	1.878826126	1.557601566
approx $y(x)$	2.00000	1.87882 1.55759	

Problem L3.4. **(E & P Exercise 2.4-12)**

The actual solution of $y' = \frac{1}{2}(y-1)^2$, $y(0) = 2$ is $y = \dfrac{x-4}{x-2}$. Apply Euler's method to produce two dot tables, as follows. The first has 101 rows, $h = 0.01$. The second has 201 rows, $h = 0.005$. Do not print the dot tables, just print the computer code that made them. Reproduce the table below and fill in the missing values. Follow the sample report for Exercise 2.4-3

`http://www.math.utah.edu/~gustafso/2250SampleProblem2.4-3.pdf`

but exclude derivation and answer check of the actual symbolic solution. Hand written work includes a check of row 2 of the first dot table ($x = 0.01$ only). Don't hand-check the second dot table. For the percentage error with $h = 0.005$, use the equation

$$Error(approx, actual) = 100\frac{|approx - actual|}{|actual|}.$$

x	0.0	0.2	0.4	0.6	0.8	1.0
y-approx, $h = 0.01$	2.0000	2.1105	2.2483	2.4250	2.6597	2.9864
y-approx, $h = 0.005$	2.0000	2.1108	2.2491	2.4268	2.6597	2.9931
actual $y(x)$	2.0000	2.1111	2.2500	2.4286	2.6597	3.0000
Error(approx,actual)	0.0000%	0.0147%	0.0400%	0.0012%	0.2619%	0.2305%

Problem L3.5. **(E & P Exercise 2.5-12)**

The actual solution of $y' = \frac{1}{2}(y-1)^2$, $y(0) = 2$ is $y = \dfrac{x-4}{x-2}$. Apply Heun's method to produce two dot tables, as follows. The first has 101 rows, $h = 0.01$. The second has 201 rows, $h = 0.005$. Do not print the dot tables, just print the computer code that made them. Reproduce the table below. Follow the sample report for Exercise 2.4-3 as in problem L3.4 above. Hand written work includes a check of row 2 of the first dot table ($x = 0.01$ only). Don't hand-check the second dot table.

x	0.0	0.2	0.4	0.6	0.8	1.0
y-approx, $h = 0.01$	2.00000	2.11111	2.25000	2.42856	2.66664	2.99995
y-approx, $h = 0.005$	2.00000	2.11111	2.25000	2.42857	2.66666	2.99999
actual $y(x)$	2.00000	2.11111	2.25000	2.42857	2.66667	3.00000
Error(approx,actual)	0.00000%	0.00005%	0.00000%	0.00006%	0.00012%	0.00033%

Problem L3.6. **(E & P Exercise 2.6-12)**

The actual solution of $y' = \frac{1}{2}(y-1)^2$, $y(0) = 2$ is $y = \dfrac{x-4}{x-2}$. Apply the RK4 method to produce two dot tables, as follows. The first has 101 rows, $h = 0.01$. The second has 201 rows, $h = 0.005$. Do not print the dot tables, just print the computer code that made them. Reproduce the table below. Follow the sample report for Exercise 2.4-3 as in problem L3.4 above. Hand written work excludes dot table checks, because the computer code is more reliable.

x	0.0	0.2	0.4	0.6	0.8	1.0
y-approx, $h = 0.01$	2.000000	2.111110	2.249998	2.428566	2.666653	2.999963
y-approx, $h = 0.005$	2.000000	2.111111	2.250000	2.428571	2.666666	2.999998
actual $y(x)$	2.000000	2.111111	2.250000	2.428571	2.666667	3.000000
Error(approx,actual)	0.000000%	0.000000%	0.000000%	0.000018%	0.000025%	0.000067%

End of Maple Lab 3: Numerical Methods.