The Basic Trial Solution Method. Outlined here is the method for a second order differential equation $ay''+by'+cy = f(x)$. The method applies unchanged for nth order equations.

Step **1**. Repeatedly differentiate the atoms of $f(x)$ until no new atoms appear. Collect the distinct atoms so found into a list of k atoms. Multiply these atoms by **undetermined coefficients** d_1, d_2, ..., d_k, then add, defining **trial solution** y.

Step **2**. Substitute y into the differential equation.

Fixup Rule I. If some variable d_p is missing in the equation, then step 2 fails. Correct the trial solution as follows. Variable d_p appears in y as term d_pA, where A is an atom. Multiply A and all its related atoms B by x. The modified expression y is called a **corrected trial solution**. Repeat step 2 until the equation contains all k variables.

Step **3**. Match coefficients of atoms left and right to write out linear algebraic equations for d_1, d_2, ..., d_k. Solve the equations for the unique solution.

Step **4**. The corrected trial solution y with evaluated coefficients d_1, d_2, ..., d_k becomes the particular solution y_p.

Symbols. The symbols c_1, c_2 are reserved for use as arbitrary constants in the general solution y_h of the homogeneous equation. Symbols d_1, d_2, d_3, ... are reserved for use in the trial solution y of the non-homogeneous equation. Abbreviations: $c =$ constant, $d =$ determined.

Superposition. The relation $y = y_h + y_p$ suggests solving $ay''+by'+cy = f(x)$ in two stages:

(a) Apply the linear equation **recipe** to find y_h.

(b) Apply **the basic trial solution method** to find y_p.

We expect to find two arbitrary constants c_1, c_2 in the solution y_h, but in contrast, no arbitrary constants appear in y_p. Calling d_1, d_2, d_3, ... *undetermined* coefficients is misleading, because in fact they are eventually *determined*.

Fixup rule II. The rule predicts the corrected trial solution y without having to substitute y into the differential equation.

- Write down y_h, the general solution of homogeneous equation $ay'' + by' + cy = 0$, having arbitrary constants c_1, c_2. Create the corrected trial solution y iteratively, as follows.

- Cycle through each term d_pA, where A is a atom. If A is also an atom appearing in y_h, then multiply d_pA and each **related atom** term d_qB by x. Other terms appearing in y are unchanged.

- Repeat until each term d_pA has atom A distinct from all atoms appearing in homogeneous solution y_h. The modified expression y is called the **corrected trial solution**.

Fixup rule III. The rule predicts the corrected trial solution y without substituting it into the differential equation. This iterative algebraic method uses the roots of the characteristic equation to create y.

- Write down the roots of the characteristic equation. Let L denote the list of distinct atoms for these roots.

- Cycle through each term d_pA, where A is a atom. If A appears in list L, then multiply d_pA and each **related atom** term d_qB by x. Other terms appearing in y are unchanged.

- Repeat until the atom A in an arbitrary term d_pA of y does not appear in list L.* The modified expression y is called the **corrected trial solution**.

*The number s of repeats for initial term d_pA equals the multiplicity of the root r which created atom A in list L.