Name KEY

Differential Equations and Linear Algebra 2250-2 Final Exam 10:30am 13 Dec 2004

Ch3. (Linear Systems and Matrices)

[50%] Ch3(a): Find the first two entries along the first row of the inverse matrix B^{-1} by the formula $B^{-1} = adj(B)/det(B)$. Evaluate determinants by any method: triangular, swap, combo, multiply, cofactor. The use of 3×3 Sarrus' rule is disallowed (2×2 use is allowed).

$$B = \left[\begin{array}{cccc} 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array} \right]$$

[50%] Ch3(b): Determine all values of k such that the system $R\mathbf{x} = \mathbf{f}$ has a unique solution [25%] and then for all such k display the solution formula for \mathbf{x} [25%].

$$R = \begin{bmatrix} 4 & 2 & -k \\ 2 & k & -2 \\ 0 & 0 & 4 \end{bmatrix}, \quad \mathbf{f} = \begin{pmatrix} 0 \\ 1 - k \\ 0 \end{pmatrix}$$

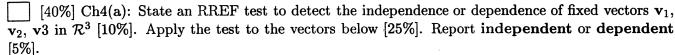
[50%] Ch3(c): Let A be an 11×11 triangular matrix with all diagonal entries equal to e^{-2} . Prove that $A\mathbf{x} = \mathbf{b}$ has a unique solution for all vectors \mathbf{b} .

(9=1,7=0, x= \frac{1}{2}-\frac{1}{2}) \tag{7} +0, Ax=b is solved by x=A-16, because New A-1 exists.

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Ch4. (Vector Spaces)



$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}.$$

- [60%] Ch4(b): Let A be a 13 × 13 matrix. Assume V is the set of all vectors \mathbf{x} such that $A^3\mathbf{x} + 2A^2\mathbf{x} + \mathbf{x} = \mathbf{0}$. Prove that V is a subspace of \mathcal{R}^{13} .
- [60%] Ch4(c): Find a basis of fixed vectors in \mathbb{R}^4 for the solution space of $A\mathbf{x} = \mathbf{0}$:

$$A = \left[\begin{array}{rrrr} 1 & 1 & -1 & 0 \\ 1 & 1 & -3 & 1 \\ 0 & 0 & -3 & 1 \\ 2 & 2 & -4 & 1 \end{array} \right]$$

[40%] Ch4(d): Find a 4×4 system of linear equations for the constants a, b, c, d in the partial fractions decomposition below [10%]. Solve for a, b, c, d, showing all **RREF** steps [25%]. Report the answers [5%].

$$\frac{x^2 + 3x + 1}{(x-1)^2(x+2)^2} = \frac{a}{x-1} + \frac{b}{(x-1)^2} + \frac{c}{x+2} + \frac{d}{(x+2)^2}$$

(a)
$$ref(ary(v_1,v_2,v_3)) = I = V_1, v_2, v_3 \text{ in deg.}$$

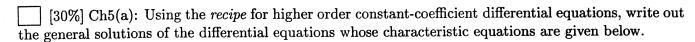
$$(\frac{1}{2}, \frac{3}{2}, \frac{2}{3}) = (\frac{3}{2}, \frac{3}{2}, \frac{3}{2}) = (\frac{3}{2}, \frac{3}{2}) = ($$

$$a = 5/q$$
, $d = -1/q$, $a = \frac{17}{27}$, $c = \frac{2}{27}$

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Ch5. (Linear Equations of Higher Order)



1.[15%]
$$(r^2 + 2r)(r+2)^2 = 0$$
,
2.[15%] $(r+2)^2(r^2+3)^2(r^2-4) = 0$

[30%] Ch5(b): Given
$$4x''(t) + 8x'(t) + 3x(t) = 0$$
, which represents a damped spring-mass system with $m = 4$, $c = 8$, $k = 3$, solve the differential equation [25%] and classify the answer as over-damped, critically damped or under-damped [5%].

[40%] Ch5(c): Determine the final form of a trial solution for y_p according to the method of undetermined coefficients. Do not evaluate the undetermined coefficients!

$$y^{iv} - 8y'' + 16y = x^2e^{2x} + \sin 2x + e^{-2x}$$

[30%] Ch5(d): Find the steady-state periodic solution for the equation

$$x'' + 2x' + 5x = 7\cos(3t).$$

(3)
$$+(r+2)^3 = 0$$
; $|y| = c_1 + (c_2 + c_3 \times + c_4 \times 2) = 2 \times (r-2)(r+2)^3 (r^2+3)^2 = 0$; $|y| = c_1 e^{2x} + (c_2 + c_3 \times + c_4 \times 2) e^{-2x} + (c_3 + c_6 \times) \cos \sqrt{3} \times f(r+c_p \times) f_m v_{3x}$
(3) $+r^2+g_r+3=0$, $(2r+3)(2r+1)=0$; $|x|=c_1e^{-3t/2}+c_1e^{-t/2}$ | $|x|=c_1e^{-3t/2}+c_1e^{-t/2}$

$$O[y] = (d_1 + d_2 \times + d_3 + c)e^{2x} \times 2 + d_5 \cos 2x + d_6 + \cos 2x + d$$

$$76(3t=x"+2x'+5x)$$

= $(-9A+6B+5A)$ $60(3t+(-9B-6A+5B))$ $60(3t+(-9B-6A+$

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Ch6.	(Eigenvalues	and	Eigenvectors))
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[30%] Ch6(a): Find the eigenvalues of the matrix A:

$$A = \left[\begin{array}{rrrr} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 2 & 2 \end{array} \right]$$

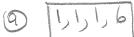
[35%] Ch6(b): Let A be a 3×3 matrix with eigenpairs

$$(1, \mathbf{v}_1), (2, \mathbf{v}_2), (3, \mathbf{v}_3).$$

Let $P = \mathbf{aug}(\mathbf{v}_3, \mathbf{v}_1, \mathbf{v}_2)$. Find a diagonal matrix D such that AP = PD [15%]. Justify your claims [20%].

[35%] Ch6(c): Assume $\det(A - \lambda I) = \det(B - \lambda I)$ for two 7×7 matrices A, B. Let A have eigenvalues 1, 2, 3, $1 \pm i$, $2 \pm \sqrt{3}i$. Find the eigenvalues of B + I, where I is the identity matrix.

[35%] Ch6(d): Give an example of a 3×3 matrix C which has eigenvalues 5, 5, 5 and three independent eigenvectors [15%]. Justify your claim [20%].



D=diag(3,1,2)

PD = AP wears \ \frac{3\forall_3 - A\forall_3}{2\forall_2 - A\forall_3}

@ [2,3,4,2±i,3±\3i

0 [C= Dig/15,5,5) [Duth B

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Ch7. (Linear Systems of Differential Equations)

	۱ [[40%]	Ch7(a).	Solve for	the ger	neral solutio	on $x(t)$.	u(t) i	n the system	below.
i	1 1	4070	Unit(a):	Solve for	the ger	nerai soruti	on $x(i)$,	$g(\iota)$ 1	n one system	DCIOW.

$$x' = x + y,$$

$$y' = -9x + y.$$

[60%] Ch7(b): Apply the eigenanalysis method to solve the system
$$\mathbf{x'} = A\mathbf{x}$$
, given

$$A = \left[\begin{array}{rrr} -3 & 1 & 1 \\ 1 & -3 & 1 \\ 0 & 0 & -3 \end{array} \right]$$

[40%] Ch7(c): Assume A is 2×2 and has eigenvalues $2 \pm 3i$. In the system $\mathbf{u}' = A\mathbf{u}$ where $\mathbf{u}(t)$ has components x(t), y(t), explain why

$$x(t) = c_1 e^{2t} \cos 3t + c_2 e^{2t} \sin 3t.$$

$$(3+3)((-3-3)^{2}-1)=0$$

$$(3+3)((-3-3)^{2}-1)=0$$

$$D = -3 - 3, -1$$

$$D =$$

(6) We solve dut(A-AI) = 2 to get A = 2 t 3i, Then by The recipe $x(t) = C_1 e^{2t} \cos 3t + c_2 e^{2t} \sin 3t$.

We know A is not transmiter, because Then A has a run eigenvalue.

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Ch10. (Laplace Transform Methods)

It is assumed that you have mem orized the basic Laplace integral table and know the basic rules for Laplace integrals. No other tables or theory are required to solve the problems below. If you don't know a table entry, then leave the expression unevaluated for partial credit.

[30%] Ch10(a): Find f(t) by partial fraction methods, given

$$\mathcal{L}(f(t)) = \frac{s+3}{(s-1)(s+2)(s+1)^2}.$$

[30%] Ch10(b): Apply Laplace's method to find a formula for $\mathcal{L}(x(t))$. Do not solve for x(t)! Document steps by reference to tables and rules.

$$x''' + x' = te^{-t} + \cos\sqrt{2}t$$
, $x(0) = 0$, $x'(0) = 0$, $x''(0) = 1$.

[35%] Ch10(c): Apply Laplace's method to the system to find a formula for $\mathcal{L}(y(t))$. Find a 2 × 2 system for $\mathcal{L}(x)$, $\mathcal{L}(y)$ [20%]. Solve it only for $\mathcal{L}(y)$ [15%]. Do not solve for x(t) or y(t)!

$$x'' = 2x + 3y,$$

 $y'' = 3x + 5y,$
 $x(0) = 0, \quad x'(0) = 1,$
 $y(0) = 1, \quad y'(0) = 0.$

[35%] Ch10(d): Solve for x(t), given

$$\mathcal{L}(x(t)) = \frac{d}{ds} \frac{s+1}{(s^2+2s+10)} + \frac{s+1}{s^2} + \frac{2+s}{s^2+4s}.$$

[35%] Ch10(e): Find
$$\mathcal{L}(f(t))$$
, given $f(t) = \frac{e^{-3t} - e^{4t}}{t}$.

(a)
$$f(x) = \frac{1}{1 + (3+1)^2 + (3+1)}$$
 $f(x) = \frac{1}{1 + (3+1)^2 + (3+1)}$ $f(x) = \frac{1}{1 + (3+1)^2 + (3+1)}$ $f(x) = \frac{1}{1 + (3+1)^2 + (3+1)}$ $f(x) = \frac{1}{1 + (3+1)^2 + (3+1)}$

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