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## Special Issue on the Mathematics of Planet Earth

*In honor of Earth Day on April 22, we present articles on food security, sustainability, resource estimation and management, sea ice modeling, and more in this **special issue**!*

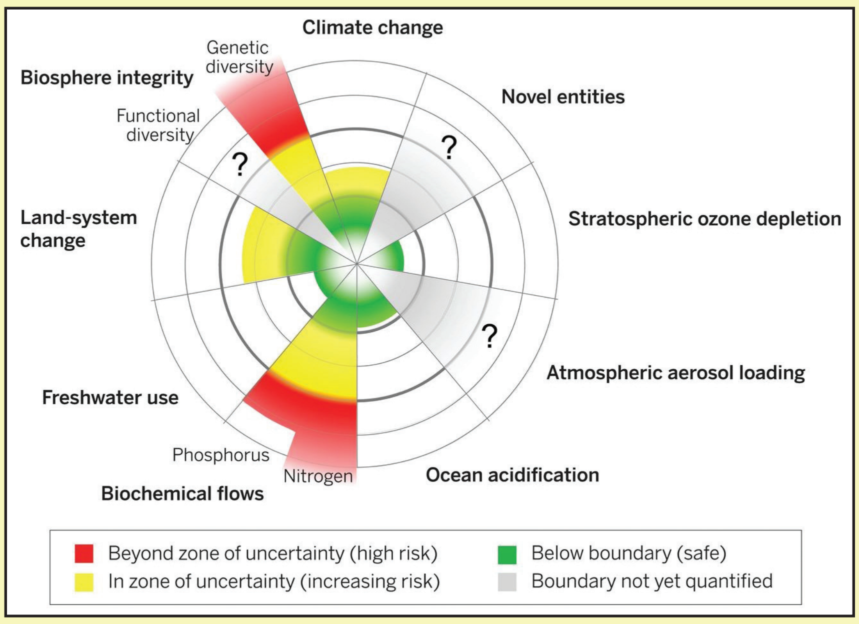


**Figure 1.** *Estimated status of the control variables for seven of the planetary boundaries. Image courtesy of Steffen et al.*

*In an article on page 5, Hans Kaper and Mary Lou Zeeman illustrate how mathematical and computational skills can help model food systems.*

# Assessing Risks to Global Food Security

## *How Mathematicians Can Help*

*By James Case*

The inaugural SIAM Conference on Mathematics of Planet Earth, held last September in Philadelphia, Pa., featured a public lecture by Molly Jahn of the University of Wisconsin, Madison (UW-Madison). Jahn, whose talk was entitled "Risks and Resilience in Global Food Systems: An Invitation for Mathematicians," holds appointments in the Department of Agronomy, the Global Health Institute, and the Center for Sustainability and the Global Environment. She has served as dean of the university's College of Agriculture and Life Sciences, director of the Wisconsin Agricultural Experiment Station, and Deputy and Acting Under Secretary of Research, Education, and Economics at the U.S. Department of Agriculture.

Jahn began her lecture by conceding that the current agricultural establishment (farmers, agribusinesses, and the agricultural research community) has been "stunningly successful" in improving agricultural productivity and efficiency. How else could we possibly be feeding a global population that has grown from under 2 billion to over 7 billion in the last century? She hastened to add, however, that the existing food delivery system is by no means ideal. It leaves some 800 million people undernourished, while 1.5 billion are overweight or obese. Meanwhile, estimates indicate that 1.4 billion tons of food are wasted each year. Though this is a small fraction of the total quantity produced, it is still significant – more than enough to feed the 1.4 billion people subsisting on $1.25 per day, or the 1.5 billion people who reside on degrading land. According to the Commission on Sustainable Agriculture and Climate Change (CSACC), more than 30 million acres of agricultural land are degraded each year due to overgrazing and other poor agricultural practices, climate change, groundwater depletion, urban sprawl, and additional human activities.

Cropland degradation, however, is not the only way in which current practices are overtaxing the planet. According to Jahn, the historic focus of research and intensive inputs

*See* **Global Food Security** *on page 4*

# Filling the Sea Ice Data Gap with Harmonic Functions

## *A Mathematical Model for the Sea Ice Concentration Field in Regions Unobserved by Satellites*

*By Courtenay Strong and Kenneth M. Golden*

Sea ice is frozen seawater that forms on the ocean surface in the Arctic basin and around the continent of Antarctica. Sea ice packs cover millions of square kilometers of our planet's surface and provide a habitat crucial to a diverse array of microorganisms, small crustaceans, marine birds, and mammals. Observed declines in sea ice amounting to approximately half a million square kilometers per decade are impacting global climate and ecosystems, and positive *sea ice-albedo feedback* is accelerating melting [2]. Sea ice has a very high albedo, meaning that it reflects most of the incoming sunlight. Declining ice coverage due to melting results in more solar energy entering the climate system, which leads to more warming and hence more melting. In fact, the September minimum of Arctic sea ice extent dropped to about 3.4 million square kilometers in 2012, which is less than half of the 1979-2000 average value of approximately 7 million square kilometers.

Since 1972, the National Aeronautics and Space Administration has been monitoring sea ice using satellites that detect the small amounts of microwave radiation emitted by the ice. The satellites detect microwave emission through clouds during both day and night, and the resulting grids at 25-km horizontal resolution provide the most spatially-complete, long-term observational record of sea ice concentrations $(0 \leq c \leq 1)$ over the polar regions in both hemispheres. Unfortunately, the orbit inclination and instrument swath of the passive microwave satellites leave a "polar data gap" around the North Pole where sea ice is not observed (see Figure 1). For many years, researchers assumed that this northernmost region of the Arctic was always covered with sea ice. However, recent precipitous losses in the polar ice pack [1] call into question this assumption, which can significantly affect overall estimates of Arctic sea ice volume. By way of anecdotal evidence, the past two Decembers (2015 and 2016) have seen freakishly warm temperatures around the North Pole, with periods of almost 50 degrees Fahrenheit above average. Such dramatic changes motivate development of an *objective* method for estimating unobserved concentrations within the gap.

We propose [6] a partial differential equation-based model with tuned stochastic spatial heterogeneity to estimate the concentrations within a region $\Omega$ on Earth's surface:

$$f(\theta,\phi) = \psi(\theta,\phi) + W(\theta,\phi),$$

where $\theta$ is longitude and $\phi$ is latitude, or $f(\vec{r}) = \psi(\vec{r}) + W(\vec{r})$, where $\vec{r} \in \Omega$. We

*See* **See Ice** *on page 3*

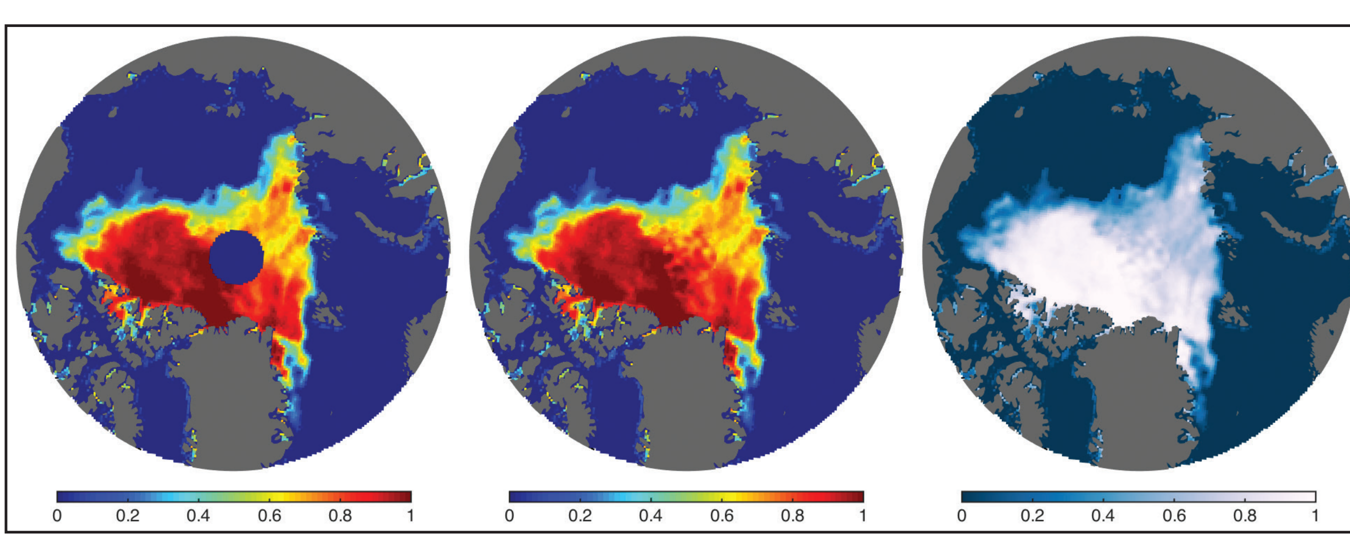


**Figure 1.** *The left image is an example of the polar data gap (dark blue disc) on August 30, 2007, with shading outside the disc indicating concentration. The middle and right images show the data fill presented here; the color shading at right is similar to that used by the National Snow and Ice Data Center (http://nsidc.org). Image adapted from [6].*

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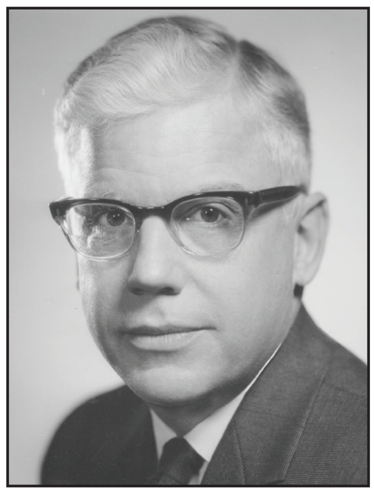
# Obituaries

*By Sarah M. Taylor, Robert J. Taylor, and Douglas W. McMillan*

Mathematician Brockway McMillan passed away in Sedgwick, Maine on December 3, 2016. Born in Minneapolis, Minn. on March 30, 1915, he was the only child of Franklin Richardson McMillan, a civil engineer, and Luvena Lucille Brockway McMillan, a school teacher. After living briefly in Philadelphia, Pa. and Brooklyn, N.Y., the McMillans returned to Minneapolis for several years, finally settling in Hinsdale, Ill. in 1925. There Brockway graduated from high school and studied for two years at the Armour Institute of Technology (which later merged with the Lewis Institute to become the Illinois Institute of Technology), before transferring to the Massachusetts Institute of Technology in 1934. He received his B.S. in 1936 and his Ph.D. in 1939, both in mathematics. His thesis, "The calculus of discrete homogeneous chaos," was supervised by Norbert Weiner.

In the fall of 1939, Brockway moved to Princeton University as a Charlotte Elizabeth Proctor Fellow; a year later he was appointed Henry B. Fine Instructor. In June 1942, a mutual friend introduced him to mathematician Elizabeth Audrey Wishard at the Institute for Advanced Study. They married in September.

Soon after his marriage, Brockway entered the Navy, where he served as an ensign at the Naval Proving Grounds in Dahlgren, Va., testing weapons and studying their ballistics. In December 1945, Brockway was reassigned to the Manhattan Project in Los Alamos, N.M., where his daughter Sarah was born. After his discharge from the Navy as a first lieutenant in 1946, Brockway joined the Mathematical Research Group at Bell Telephone Laboratories in Murray Hill, N.J. His son Douglas was born in 1947 in nearby Summit, where—except for two assignments with the federal government in Washington, D.C.—the McMillans lived until 1979. His son Gordon was born in Boston, Mass. in 1952 while Brockway attended the Lincoln Summer Study Group. In Summit, Brockway served on the Board of Education, eventually becoming Board president.



*Brockway McMillan, 1915-2016.*

At Bell Labs, Brockway's research produced papers and theorems on information theory, in collaboration with Claude Shannon and John Tukey. Other technical interests included electrical network theory and random processes. In later years he fondly recalled his time at Murray Hill, the intellectual stimulation of like-minded colleagues, their noontime experiments with boomerangs and word games, singing in the Murray Hill Chorus, and playing the "wobble organ"[1]—a DIY electronic musical instrument invented by Larned Meachem, Brockway's Bell Labs colleague. In 1955, Brockway left the research group to become Assistant Director of Systems Engineering and, in 1959, Director of Military Research.

By the late 1950s, Brockway's expertise in communication systems research and development was in demand at the National Security Agency and the Department of Defense. During the winter of 1958-59, Brockway served as assistant to James Killian, President Dwight D. Eisenhower's science advisor. In 1961, President John F. Kennedy appointed Brockway as Assistant Secretary of the Air Force for Research and Development. Two years later, he became Under Secretary of the Air Force and concurrently the second director of the National Reconnaissance Office (NRO). As director, he advocated maintaining the NRO as the primary U.S. agency in space reconnaissance, and presided over the development of a second-generation, high-resolution imaging satellite system.

Brockway returned to Bell Labs in 1965, serving as Vice President for Military Systems from 1969 until his retirement. In 1967, the McMillans bought an 1820s farmhouse overlooking the Benjamin River in Sedgwick, where they summered regularly until retiring there in 1979.

During retirement, Brockway continued his research and correspondence with fellow scientists and mathematicians, consulted for the U.S. government and Eastern Airlines, and stayed active in the American Mathematical Society and SIAM. He served as president of SIAM from 1959 to 1960. Brockway was a fellow of the Institute of Electrical and Electronics Engineers, a fellow of the American Association for the Advancement of Science, and a member of the National Academy of Engineering.

He and Audrey traveled extensively in the U.S. and Europe, spending one winter in Berkeley, Calif., to give a series of invited lectures at the University of California. At home in his Sedgwick darkroom, Brockway developed and printed thousands of pictures, which he exhibited locally. He served as president of the Sedgwick-Brooklin Historical Society and chairman of the

*See* **Obituaries** *on page 5*

[1] http://120years.net/the-wobble-organ-larned-ames-meacham-usa-1951/

## Sea Ice

*Continued from page 1*

suggest prescribing the scalar field $\psi$ to be a solution of Laplace's equation

$$\Delta\psi = 0,$$

in spherical coordinates with boundary conditions taken from observations on the boundary $\partial\Omega$ of the polar data gap. A unique solution for $\psi$ exists if $\partial\Omega$ is sufficiently smooth and the concentration is a continuous function along $\partial\Omega$. One can numerically obtain this solution by expressing the Laplacian as a second-order finite difference operator. The stochastic term $W$ provides realistic deviations from $\psi$, and was tuned by collecting samples $W_s$ of the difference between observed concentrations and $\psi$ in three circular regions, $C_j,\ j=1,2,3,$ around the polar data gap,

$$W_s(\vec{r}) = f_{\text{obs}}(\vec{r}) - \psi(\vec{r}),$$
$$\vec{r} \in C_j, \quad j = 1,2,3,$$

where $f_{\text{obs}}$ denotes observed concentrations. Based on analysis of thousands of samples, we formulate a seasonally varying amplitude for $W$ and introduce realistic spatial autocorrelation by convolution of spatially uncorrelated noise with a Gaussian function. Figure 1 (on page 1) shows an example of this model applied to the polar data gap in map view for August 30, 2007. Figure 2 below shows this same example with concentrations represented by a third vertical dimension. Tests in regions around the polar data gap reveal observation-model correlations of 0.6 to 0.7 and absolute deviations of order $10^{-2}$ or smaller.

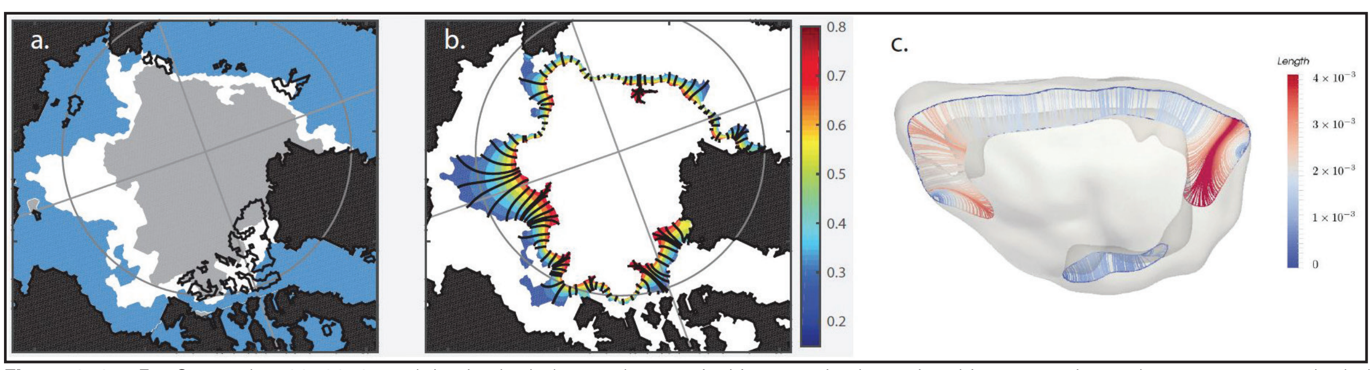



**Figure 3. 3a.** *For September 29, 2010, pack ice is shaded gray, the marginal ice zone is shown in white, sparse ice and open ocean are shaded blue, land is shaded black, and islands over which concentrations were interpolated are outlined in black. Image adapted from [7].* **3b.** *The solution $\psi$ to Laplace's equation within the marginal ice zone (MIZ) is shaded, and the black curves are examples of streamlines through $\psi$ whose arc lengths define MIZ width. Image adapted from [7].* **3c.** *Colored curves are examples of streamlines of a solution to Laplace's equation on a cross-section of the cerebral cortex of a rodent brain, the arc lengths of which are used to objectively measure cortical thickness. Image adapted from [4].*

Our formulation of the data fill was motivated by our prior work [5] using Laplace's equation to approximate sea ice concentrations within the marginal ice zone (MIZ), the region where sea ice concentrations transition from dense pack ice $(c \geq 0.8)$ to open ocean $(c \leq 0.15)$ (see Figure 3a). The MIZ is important from both climatic and ecological perspectives, and is characterized by strong ocean-ice interactions where waves penetrate the sea ice pack. By adapting medical imaging techniques for measuring non-convex shapes and volumes in the human body [3], we define the width of the MIZ as the arc length of a streamline through the solution to Laplace's equation (see Figure 3b). Spatially averaging the widths reveals a dramatic 39% widening of the MIZ over the satellite record [7]. Figure 3c illustrates an example of the Laplace method applied to measuring the thickness of a rodent cerebral cortex.

In the formulation for filling the polar data gap, a least-squares linear function could replace the solution to Laplace's equation, but at the expense of observation-model agreement along $\partial\Omega$. One could think of the function $\psi$ more generally as the solution to a Poisson equation, or a more general elliptic equation incorporating a local conductivity or diffusivity $D(\vec{r})$,

$$\nabla \cdot (D \nabla \psi) = 0,$$

thereby accommodating local extrema precluded by Laplace's equation. By further increasing the complexity and computational expense, the polar data gap could also be filled by sophisticated numerical models of sea ice evolution, which incorporate dynamics and thermodynamics. In any event, developing methods to objectively fill this critical data gap is a worthy mathematical challenge and will impact our understanding of Earth's rapidly-changing climate.

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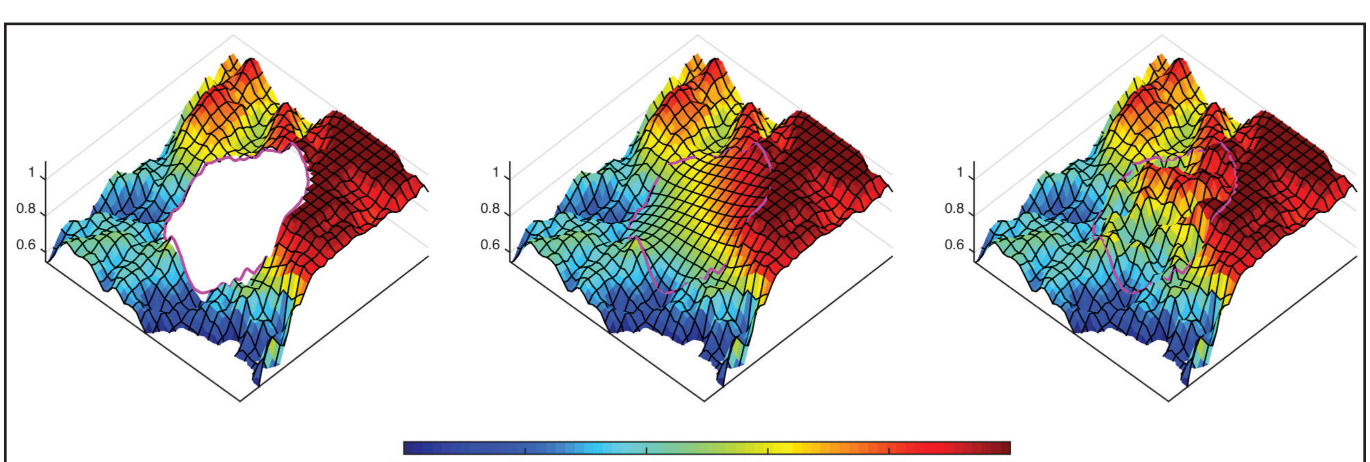


**Figure 2.** *The example in Figure 1 (on page 1) with concentration indicated by shading and surface elevation. Panels show the polar data gap (left), the solution to Laplace's equation within the gap (middle), and the solution with realistic spatial heterogeneity added (right). Image adapted from [6].*

*Courtenay Strong is an associate professor of atmospheric sciences at the University of Utah. A substantial component of his research focuses on modeling and analysis of the cryosphere, which includes sea ice and snow. Kenneth M. Golden is a distinguished professor of mathematics and an adjunct professor of bioengineering at the University of Utah. His research is focused on developing mathematics of composite materials and statistical physics to model sea ice structures and processes.*