MATH 5610/6860
HOMEWORK #4, DUE WED OCT 5

Note: Sample code is provided in the class website.

1. B&F 2.6.4 a,b (Müller's method). Please specify the initial iterates z_0, z_1, z_2 you used to find a particular root and the number of iterations.
2. K&C 6.2.23. The polynomial

$$p(x) = 2 - (x+1) + x(x+1) - 2x(x+1)(x-1)$$

interpolates the first four points in the table

x	-1	0	1	2	3
y	2	1	2	-7	10

By adding one additional term to p, find a polynomial that interpolates the whole table.

3. B&F 3.1.6 a,b, additionally implement the divided differences algorithm 3.2 to obtain the same results. Then compare the actual interpolation error to a bound derived from Theorem 3.3 (i.e. B&F 3.1.8 a,b). Produce a plot showing the function and the interpolation polynomials of degrees 1, 2 and 3 similar to the one provided in the class website.
4. K&C 6.2.18 Prove that if f is a polynomial, then the divided differences $f[x_0, x_1, \ldots, x_n]$ is a polynomial in the variables $x_0, x_1, \ldots, x_n$. (i.e. if we freeze all the x_j for $j \neq i$, then $q(x_i) \equiv f[x_0, x_1, \ldots, x_n]$ is a polynomial in x_i). You may prove this by an induction argument:
   i. Prove that $f[x_0]$ is a polynomial in x_0.
   ii. Prove that $f[x_0, x_1]$ is a polynomial in x_0 and x_1. Because nodes in divided differences can be permuted, it is enough to show that $f[x_0, x]$ is a polynomial in x. This step is not needed for induction argument but it helps to figure out how to do the general case.
   iii. Assuming $(n-1)$-th order divided differences are polynomials of their variables, prove the statement for n-th order divided differences. Again because the order of the nodes in divided differences does not matter, it is enough to show that $f[x_0, x_1, \ldots, x_{n-1}, x]$ is a polynomial. (Hint: use recursion formula B&F eq. (3.9))