On optimal design of transient electromagnetic waveforms

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Summary

The resolution of electromagnetic imaging is optimized with respect to the transmitter waveform. This problem is formulated as a generalized eigenvalue problem for the intensity of the current injected at the transmitting source which is solved numerically. As an example we consider transmitter waveform design to maximize the response of an anisotropic layer. The optimal excitation is a sinusoidally damped oscillating current, which shows tremendous increase in resolution of the anisotropic layer compared to the fields generated by a conventional time domain transmitter waveform.

Introduction

This paper deals with the problem of optimization of the resolution of the transient electromagnetic method of exploration by designing a transmitter waveform which is optimal for detecting the particular target. The approach which we adopt has been previously discussed with regards to direct current measurements in [1, 2]. We have shown in [3, 4] that inversion algorithms using such optimized data can be especially effective. The problem is related to the problem of optimizing an antenna array for transmission thorough the atmosphere [6, 8, 10].

Formulation of the problem

We assume that a transient electric current can be injected at the transmitter site, while the time dependent magnetic field is measured at some receiver point. The current j injected at the transmitter point x_0 is $j = j(t)$. The measured horizontal component of the magnetic field is $h_x(t)$ at the receiver point x_1. In the numerical example below we will assume that the transmitter is located at the surface of the earth, and the receiver is in a borehole.

The measured field $h_x(t)$ is compared with the field $h_x^0(t)$ generated by the same current for some reference or background medium of known conductivity. We want to find the function $j(t)$ which maximizes the difference in the fields, $\delta h_x(t) = h_x(t) - h_x^0(t)$. Since the problem is homogeneous, we relate the measured difference to the measured background response. We will show that this is equivalent to restricting the power spent for the injection of the currents by the same value.

Let the earth material be characterized by the magnetic permittivity μ_0, and an electric conductivity σ. We assume that the background medium is a homogeneous earth with the conductivity σ_0. Maxwell's equations governing the electromagnetic field in the earth are:

$$\nabla \times e + \frac{\partial b}{\partial t} = 0 \ , \quad \nabla \times h + j = 0 \ , \qquad (1)$$

with linear constitutive relations:

$$\sigma e = j \ , \ \text{ and } \mu h = b \ . \qquad (2)$$

The generated magnetic field in the real medium, h_x, is given by a linear operator $\mathcal{H}$, $\mathcal{H} = \mathcal{H}(\sigma)$, applied to the current j. The same current injected in the background medium gives rise to the field h_x^0 described by an operator $\mathcal{H}_0$, $\mathcal{H}_0 = \mathcal{H}_0(\sigma_0)$:

$$h_x = \mathcal{H} j \ , \ \ h_x^0 = \mathcal{H}_0 j \ , \qquad (3)$$

Suppose, that we measure the received signal in a time interval from 0 to T. An increase of the resolution is achieved if we can increase the values of the measured anomalous difference in comparison with the background response. An appropriate functional can be introduced as a ratio of the L_2 norm of the measured anomalous difference to the corresponding norm of the background signal.

Hence, the problem of maximizing the resolution of the perturbation $\sigma - \sigma_0$ can be formulated as the following optimization problem:

$$||h_x - h_x^0|| = ||\mathcal{H} j - \mathcal{H}_0 j|| \to \max_{j,\, ||\mathcal{H}_0 j|| = 1} \ . \qquad (4)$$

Here the norm of a function f is taken to be

$$||f|| = \int_0^T f^2(t)\, dt \ . \qquad (5)$$

Solving problem 4 is equivalent to solving

$$\frac{||\mathcal{H}j - \mathcal{H}_0 j||}{||\mathcal{H}_0 j||} \to \max_j \ . \qquad (6)$$

this problem can be reduced to the generalized eigenvalue problem and solved numerically.

Solution of the problem for a discretized transient signal

The signal shape which maximizes the measured response field of the conductivity anomaly is obtained as a solution of the optimization problem (4).
Assume that the time interval $[0, T]$ is discretized using $n+1$ points in small intervals of a length T/n, and the function $j(t)$ is constant in each interval:

$$j(t) = j_k, \ t_{k-1} < t \leq t_k, \ k = 1, 2, ...n. \qquad (7)$$

Let J be the vector with the components $j_k, \ k = 1, 2, ...n$.
The linear operators $\mathcal{H}$ and $\mathcal{H}_0$ are defined if we describe their action on the unit pulse at the moment t_1. This pulse excitation corresponds to the vector $P = [1, 0, ..., 0]^T$. Suppose that the vectors of the discretized measured and the background fields are $h_P = \{h_P(t_k)\}, k = 1, ..., n$, and $h_P^0 = \{h_P^0(t_k)\}, k = 1, ..., n,$. From (3) it follows that

$$h_P = \mathcal{H}P \ , \ \ h_P^0 = \mathcal{H}_0 P \ , \qquad (8)$$

and it follows from the special structure of the vector P that the first column of the matrix of the operator $\mathcal{H}$ is equal to the vector h_P, and, similarly, the first column of the matrix of the operator $\mathcal{H}_0$ is equal to the vector h_P^0.
The reaction of the system to the unit pulse at the time point t_2, and, generally t_k, is merely a shifted vector h_P, and, respectively, h_P^0. Hence the matrices H and H_0 of the operators $\mathcal{H}$ and $\mathcal{H}_0$ are compounded from the shifted vectors h_P and h_P^0 respectively. The $k - th$ column of the matrix H is given by the vector $[0, ..., 0, h_P(t_1), h_P(t_2), ...]^T$, in which the first $k-1$ entries are zeroes. Similarly, the $k-th$ column of the matrix H_0 is given by the vector $[0, ..., 0, h_P^0(t_1), h_P^0(t_2), ...]^T$. Now the action of the operators on an arbitrary vector J or the fields due to an arbitrary discretized excitation J can be easily calculated as vectors HJ for the real medium and $H_0 J$ for the background medium.
The optimization problem (4) now can be formulated:

$$\frac{||HJ - H_0 J||^2}{||H_0 J||^2} \to \max_{J \in R^n} \ , \qquad (9)$$

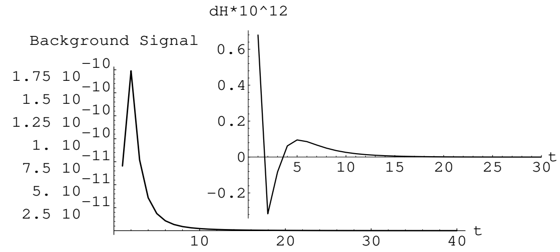

Figure 1: The background response H and the anomalous response dH due to the unit pulse.

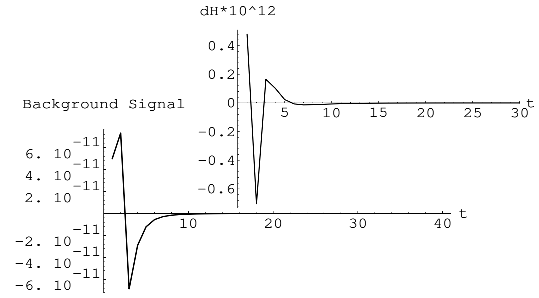

Figure 2: The background response H and the anomalous response dH due to the signal $f(t)$.

where we assume the Euclidian norm of the vectors. Using a variational approach we reduce this problem to the following generalized eigenvalue problem:

$$(H - H_0)^T (H - H_0) J - \lambda H_0^T H_0 J = 0 \qquad (10)$$

The first eigenvector solving this problem is the excitation which maximizes the ratio (9).
Exploiting the symmetry of the matrices $(H - H_0)^T (H - H_0)$ and $H_0^T H_0$, we can reduce the solution of the problem (10) to an eigenvalue problem. Introducing the vector $Y = H_0 J$, so that $J = H_0^{-1} Y$ the problem takes the form:

$$(H_0^T)^{-1} (H - H_0)^T (H - H_0) H_0^{-1} Y - \lambda Y = 0, \ (11)$$

which is an eigenvalue problem, and can be solved using standard techniques.

Numerical simulation

In order to demonstrate the efficacy of the developed approach in improving the resolution of electromagnetic imaging, we consider a survey to determine possible anisotropy of a thin layer embedded in a homogeneous earth. This model was used in [7] in a separate study of transmitter waveform optimization.

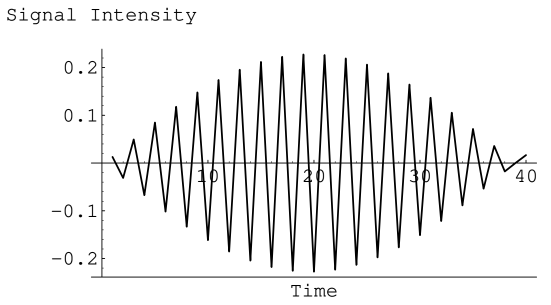

Figure 3: The shape of the optimal excitation.

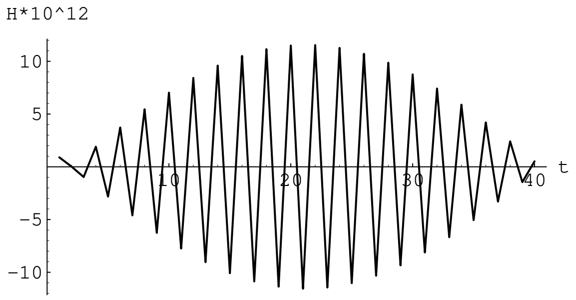

Figure 4: The background response H due to the optimal signal.

The transmitter is located on the surface of the earth, while the receiver is placed in the borehole. The measured response is compared with the response of a similar isotropic layer, which is called the background response H. The anomalous response dH is the difference between the real measured response from the anisotropic layer and the background response.
Figure 1 shows the response of this isotropic layer when the applied excitation is the unit pulse. Also shown there is the anomalous response dH, or the difference between the real measured and the backgound responses. The data are calculated in [7] using a forward modeling integral equations algorithm [11]. We use the data due to this signal for the comparison with the response generated by the optimal current. Also for comparison we use data due to the signal resembling some features of the optimized signals constructed in [7]. This signal $f(t)$ is constructed as a positive pulse at the moment t_1 and then a negative pulse at the moment t_2, so that $f(t) = 1/\sqrt{(2)}P, t = t_1,\ f(t) = -1/\sqrt{(2)}P, t = t_2,\ f(t) = 0, t = t_3, ...t_n$. The intensity of all currents is assumed to be unit, which generates normalizing coefficients in the formula for $f(t)$. The background and anomalous responses due to this signal are shown in Figure 2.

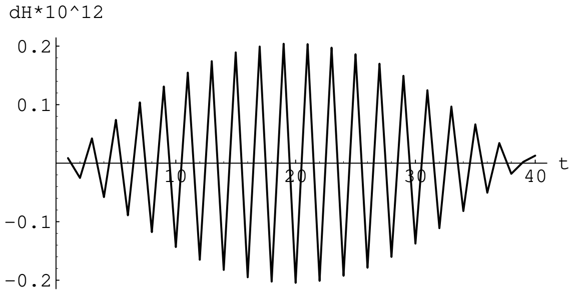

Figure 5: The anomalous response dH due to the optimal signal.

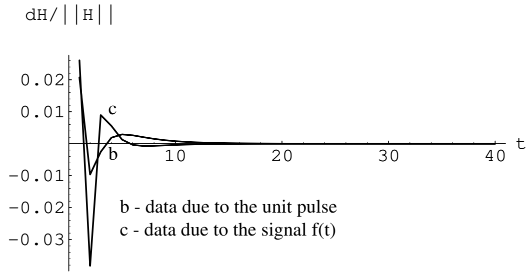

Figure 6: The normalized measured difference: $dH/||H||$ generated by different transmitter excitations.

Figure 3 shows the shape of the optimal excitation calculated using the technique developed above. The optimal source is designed specifically for the model under consideration. Exploiting the described approach we reduce the generalized singular value problem to the standard SVD, and use a standard subroutine from LINPACK [9] to solve it. This optimal signal is an oscillating current with a sinusoidally damped intensity. It can be generated by switching positive and negative pulses of time varying intensity. Figures 4 and 5 show the background response and the response difference due to this signal.
Figure 6 shows the normalized measured difference $dH/||H||$ for the unit pulse and for the signal $f(t)$, while Figure 7 shows that due to the optimal current. Figures 8 and 9 show the ratio dH/H of the response difference to the background response calculated at each point of the time interval. Comparison of the curves in Figures 6 and 8 show that the signal $f(t)$ generates a larger anomalous response and a larger response ratio than does the pulse excitation. However, comparison of the functions on Figures 7 and 9 demonstrate the optimality of the transmitter current

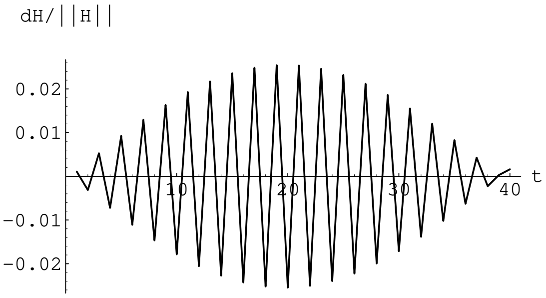

Figure 7: The normalized measured difference: $dH/||H||$ due to the optimal current.

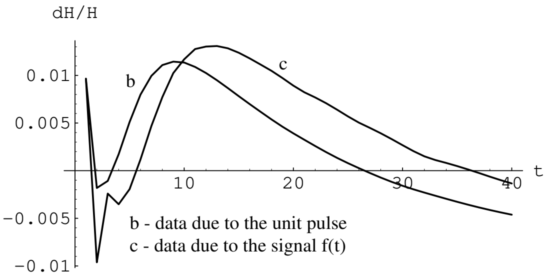

Figure 8: The ratio dH/H of the anomalous response to the background response calculated at each point of the time interval for different transmitter excitations.

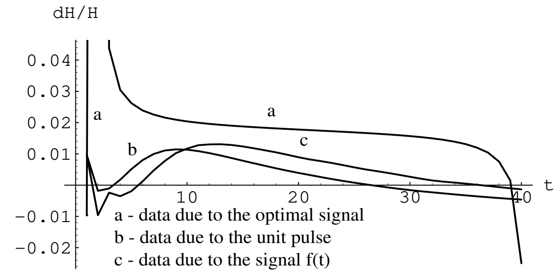

Figure 9: The ratio dH/H of the anomalous response to the background response for the optimal current in comparison with similar ratios for other transmitter excitations.

waveform designed using developed here approach.

Conclusions

The method of optimal design of time dependent excitations is developed for increasing the resolution of electromagnetic imaging. These optimal current patterns maximize the ratio of the anomalous response to the background response. The optimal time dependent signal is constructed for detecting anisotropy of a thin layer embedded in a homogeneous earth.

Acknowledgement

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References

[1] E. Cherkaeva and A. C. Tripp, Optimal survey design using focused resistivity arrays, IEEE, Trans. Geoscience and Remote Sensing, 34 (1996), 2, 358–366.

[2] E. Cherkaeva and A.C. Tripp, On the resolution of geoelectric imaging, submitted.

[3] E. Cherkaeva and A. C. Tripp, Inverse conductivity problem for inexact measurements, Inverse Problems, 12 (1996), 869–883.

[4] E. Cherkaeva and A. C. Tripp, Source optimization in the inverse geoelectrical problem: in Inverse Problems in Geophysical Applications, eds. H.W. Engl, A. Louis, and W. Rundell, SIAM, 1996.

[5] R.F. Harrington, Field computation by moment methods: New York, Macmillan, 1968.

[6] B. Kriegshauser, 1997, Experimental design for time domain surface to borehole electromagnetic applications: PhD Dissertation, University of Utah.

[7] A. Kirsch and P. Wilde, The optimization of directivity and signal-to-noise ratio of an arbitrary antenna array, Math. Meth. Appl. Sci., 1988, v.10, 153-164.

[8] J.J. Dongarra, J.R. Bunch, C.B. Moler, G.W. Stewart, "LINPACK Users' Guide." SIAM, 1979.

[9] D. Pozar, D.H. Schaubert, and R.E. McIntosh, The optimum transient radiation from an arbitrary antenna, IEEE Trans. Antennas and Propag., 1984, v. AP-32, 6, 633-640.

[10] Z. Xiong, 1992, Electromagnetic modeling of three-dimensional structures by the method of system iterations using integral equations, Geophysics, 57, 1556-1561.