PCMI Project:
Resetting Reentrant Excitation Oscillations in Different Geometries

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motivation...

• normal rhythm is set by cells in SA node
• excitation spreads via conduction system
• cardiac tissue is excitable...
  ★ large enough perturbation from rest
    ⇒ action potential
    ⇒ refractory period

• Reentrant Tachycardia...
  ★ a conduction pathway loops onto itself → self-sustained oscillation of excitation
  ★ rapid heart rate determined by time it takes excitation to travel around the loop

• Goal...
  ★ deliver stimulus to area, utilizing refractory properties, to annihilate tachycardia
mechanism for annihilation of tachycardia...

- consequences of refractory property of 1D excitable media...
  - a stimulus delivered...
    - ✯ early in the refractory period will produce 0 new waves
    - ✯ after the refractory period will produce 2 new waves
    - ✯ in just the right spot will produce only 1 new wave
  - two waves colliding head on will be annihilated by the refractory periods

- simple picture of tachycardia....
  - represent the loop in the conduction system as a 1D ring of excitable media
  - the tachycardia is a traveling wave of excitation on the 1D ring
  - the above observations provide a mechanism for both
    the annihilation and phase resetting of tachycardia
    - ✯ can create phase resetting curves to characterize and study the behavior
the project...

- Goal: to explore this phenomena on different geometries
  - ring with a tail
  - ring with a chord

- Fitzhugh-Nagumo was the suggested model for excitability
  - boundary conditions will be challenging for complicated geometries
  \[ \Rightarrow \text{ so, we took a different approach in modeling excitability...} \]
Red-Blue-White model of excitation...

★ a number of discrete cells on a ring
★ each cell can be in one of four states
★ the state of each cell is updated every discrete time step
★ each state is assigned an arbitrary numerical value
  - resting: 0
  - excited: +10
  - absolutely refractory: $-\infty$
  - relatively refractory: $-c \ldots -1$

★ input variables...
  $b =$ length of absolute refractory period
  $c =$ length of relative refractory period
  $N =$ how many cells on a ring

★ show movie of pulse on a ring with $N = 50$, $b = 5$, $c = 5$
adding a stimulus...

- $s =$ strength of the stimulus (note: $s \geq 1$)
- $t_s =$ time, or phase, at which the stimulus is applied to the ring
- if the stimulus is delivered to...
  (i) absolutely refractory cell $\rightarrow$ no response
  (ii) resting cell $\rightarrow$ 2 new waves propagate in opposite directions
  (iii) relatively refractory cell...
    * early in the relative refractory period $\rightarrow$ no response
    * late in the relative refractory period $\rightarrow$ 1 new wave propagates back

* show movies for various $t_s$ with $N = 50, b = 5, c = 5, s = 4$, and threshold=$+1$
phase resetting curve...

- $T_0 =$ time it takes the original pulse to travel around the ring once
- $\phi_s =$ phase of the stimulus (with respect to a reference cell)
  - $\phi_s = 1/N \Rightarrow$ stim is applied to ref cell when the orig pulse is also at that point
  - $\phi_s = 1/2 \Rightarrow$ stimulus is applied when original pulse is halfway around the ring
  - $\phi_s = 3/4 \Rightarrow$ stimulus is applied when original pulse is a $\frac{3}{4}$ the way around
  - $\phi_s = (N + 1)/N \Rightarrow$ same result as $\phi_s = 1/N$
- if a stimulus is applied at a given phase...
  - $T_1 =$ is the time since the ref point last saw a pulse to when it sees another
  - in general..
    - $T_j =$ is the time at which the ref point has seen another $j$ pulses
phase resetting curve...

For a given $N$, $b$, $c$, and $s$, stimulate the reference cell at various phases, and calculate $T_1$.

We get the same type of curves generated by Fitzhugh-Nagumo excitability (Glass, 1995).
so what...

- why did we take this approach...
  - wanted to develop a simple way to observe the behavior found in FN
    - annihilation and phase resetting
    - reproduce the phase resetting curves
  - modify this simple code for different for geometries
  - perhaps examine periodic stimulation
  - then, moving into a continuous PDE model, we might know what to look for
  - of course, there might be more interesting behavior in the PDE model, but...
  - this simple discrete model is a good way to investigate the complicated problem
    and gets us (me) thinking about the behavior of excitable media in general,
    and the corresponding rules that govern its behavior