

## Math 5710. Extra problem set 2

**A.** Let  $X$  be a real inner product space with induced norm  $\|x\| = \langle x, x \rangle^{1/2}$ . Use the properties of the inner product to prove:

1. *Pythagorean Theorem.* For every  $x, y \in X$  with  $\langle x, y \rangle = 0$ ,

$$\|x + y\|^2 = \|x\|^2 + \|y\|^2.$$

2. *Polarization Identity.* For every  $x, y \in X$ ,

$$\langle x, y \rangle = \frac{1}{4}(\|x + y\|^2 - \|x - y\|^2).$$

3. *Parallelogram Law.* For every  $x, y \in X$ ,

$$\|x + y\|^2 + \|x - y\|^2 = 2(\|x\|^2 + \|y\|^2).$$

**B.** Use the Gram-Schmidt process to generate the first five orthogonal Legendre polynomials  $\phi_0, \phi_1, \dots, \phi_4$  on the interval  $[-1, 1]$ , using the inner product  $\langle f, g \rangle = \int_{-1}^1 f(x)g(x) dx$ .