

Chapter 1 Problems

5. If the sample space has n elements, then \mathcal{F} has 2^n elements: Recall that \mathcal{F} is the collection of all subsets of S , and there are 2^n subsets of a set of n distinct elements. Since 6 is not 2^n for any integer $n \geq 1$, this answers the question.

7. There are other, equivalent, ways of writing these answers. It might help to draw Venn diagrams in each case.

(a) $A^c \cup B^c \cup C^c$.

(b) $(A \cap B \cap C^c) \cup (A^c \cap B \cap C) \cup (A \cap B^c \cap C)$.

(c) $(A \cap B \cap C)^c$.

(d) $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$.

9. I will follow your text's notation, from here on, and write AB instead of $A \cap B$, etc.

(a) The probability of exactly four heads is $1/2^4 = 1/16$. Therefore, the probability of at least three heads is

$$\Pr\{\text{exactly three H}\} + \Pr\{\text{exactly four H}\} = \frac{1}{4} + \frac{1}{16} = \frac{5}{16}.$$

(b) The probability of exactly three heads is

$$\begin{aligned} & \Pr(H_1 H_2 H_3 T_4) + \Pr(H_1 H_2 T_3 H_4) + \Pr(H_1 T_2 H_3 H_4) \\ & \qquad \qquad \qquad + \Pr(T_1 H_2 H_3 H_4) \\ &= \frac{1}{2^4} + \frac{1}{2^4} + \frac{1}{2^4} + \frac{1}{2^4} \\ &= \frac{4}{2^4} \\ &= \frac{1}{4}. \end{aligned}$$

(c) We seek to find

$$\Pr(H_1 H_2 H_3 H_4) + \Pr(H_1 H_2 H_3 T_4) + \Pr(T_1 H_2 H_3 H_4) = \frac{3}{16}.$$

10. The chances are $35/36$ that a double six fails to appear in one toss. By independence, the probability of failure in 24 trials is $(35/36)^{24} \approx 0.5086$.

11. The roots of this quadratic polynomial are

$$\frac{-S_1 \pm \sqrt{S_1^2 - 4S_2}}{2}.$$

These are real roots if and only if $S_1^2 \geq 4S_2$. In order to obtain $S_1^2 \geq 4S_2$, (S_1, S_2) must be equal to one of the following: $(2, 1); (3, 1); (3, 2); (4, 1); (4, 2); (4, 3); (4, 4); (5, 1); \dots; (5, 6); (6, 1); \dots; (6, 6)$. There are 19 such possibilities; therefore, $\text{Prob} = 19/36 \simeq 0.528$.

22. The true answers are: (a); (b); (e); and (f). Answers (c) and (d) are false.