Reading and Problem Assignment #9
Math 501–1, Spring 2006
University of Utah

Read Chapter 7, sections 7.1–7.4, and review conditional expectation by reading through section 7.5 (edition 7).

The following are mainly borrowed from your text.

Problems:

1. A player throws a fair die and simultaneously flips a fair coin. If the coin lands heads then she wins twice, and if tails, then one-half of the value that appears on the die. Determine her expected winnings.

2. If X and Y are independent uniform-$(0\,,1)$ random variables, then prove that

$$E\left(|X-Y|^{\alpha}\right)=\frac{2}{(\alpha+1)(\alpha+2)}\qquad\text{for all } \alpha>0.$$

3. A group of n men and n women are lined up at random.
(a) Find the expected number of men who have a woman next to them.
(b) Repeat part **(a)**, but now assume that the group is randomly seated at a round table.

4. Let $X_1, X_2, \ldots$ be independent with common mean μ and common variance σ^2. Set

$$Y_n = X_n + X_{n+1} + X_{n+2}\qquad\text{for all } n\geq 1.$$

Compute $\mathrm{Cov}(Y_n\,,Y_{n+j})$ for all $n\geq 1$ and $j\geq 0$.

Theoretical Problems:

1. Suppose X is a nonnegative random variable with density function f. Prove that

$$E(X)=\int_0^\infty P\{X>t\}\,dt. \tag{eq.1}$$

Is this still true when $P\{X<0\}>0$? If "yes," then prove it. If "no," then construct an example.

2. (Hard) Suppose $X_1,\ldots,X_n$ are independent, and have the same distribution. Then, compute $\phi(x)$ for all x, where

$$\phi(x) := E\left[\,X_1 \mid X_1+\cdots+X_n = x\right].$$