# Reading and Problem Assignment #6
## Math 501–1, Spring 2006
## University of Utah

Midterm preparation. Read **all** of Chapter 5 (continuous random variables) **except** the section, “The distribution of a function of a random variable.”

The following are borrowed from your text.

**Problems:**

**1.** Let $X$ be a random variable with density function

$$f(x) = \begin{cases} c(1-x^2) & \text{if } -1 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

**(a)** What is $c$?
**(b)** Compute the distribution function $F$.
**(c)** Calculate $P\{0 < X < 1.5\}$.
**(d)** Compute $EX$ and $\text{Var}X$.

**2.** Suppose $X$ is normally distributed with mean $\mu = 10$ and variance $\sigma^2 = 36$. Compute:
**(a)** $P\{X > 5\}$.
**(b)** $P\{4 < X < 16\}$.

**3.** Let $X$ be uniformly distributed on $[0\,,1]$. Then compute $E[X^n]$ for all integers $n \geq 1$. What happens if $n = -1$?

**4.** The density function of $X$ is

$$f(x) = \begin{cases} a + bx^2 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

We know that $EX = \frac{3}{5}$. Compute $a$ and $b$.

**Theoretical Problems:**

**1.** Let $X$ have the exponential($\lambda$) distribution, where $\lambda > 0$ is fixed. That is, we suppose that the density function of $X$ is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Prove that for all integers $k \geq 1$,

$$E[X^k] = \frac{k!}{\lambda^k}.$$

(`Hint:` Use gamma functions.)

**2.** Let $X$ have the exponential($\lambda$) distribution, where $\lambda > 0$ is fixed. Then, compute $P(X > x + y \,|\, X > y)$ for all $x, y > 0$. Use this to prove that for all $x, y > 0$,

$$P(X > x + y \,|\, X > y) = P\{X > x\}.$$

This property is called “memoryless-ness.”