

Reading and Problem Assignment #3
Math 501–1, Spring 2006
University of Utah

Read chapter 3 (conditional probability and independence).

The following are borrowed from your text.

Problems:

1. What is the probability that at least one of a pair of fair dice lands on 6, given that the sum of the dice is i , for $i = 2, 3, \dots, 12$?
2. The king comes from a family of 2 children. What is the probability his sibling is a girl?
3. Suppose that 5% of all men and 0.25% of all women are colorblind. A colorblind person is chosen at random. What is the probability that this person is male? Assume that there are an equal number of men and women. What if the population consisted of twice as many men as women?
4. Suppose we have 10 coins such that if the i th coin is flipped then heads will appear with probability $i/10$ ($i = 1, \dots, 10$). One of the coins is selected at random, and then flipped. We are told that it shows heads. What is the probability that it was the fifth coin?
5. Let $S = \{1, \dots, n\}$ and suppose that A and B are two independent, random subsets of S , all subsets being equally likely.
 - (a) Prove that $P\{A \subset B\} = (3/4)^n$.
[Hint: Use Bayes' formula, and condition on the number of elements of B .]
 - (b) Prove that $P\{A \cap B = \emptyset\} = (3/4)^n$.

Theoretical Problems:

1. In Laplace's rule of succession (page 109, seventh edition), suppose that the first n flips resulted in r heads and $(n - r)$ tails. Show that the probability that the $(n + 1)$ st flip turns up heads is $(r + 1)/(n + 2)$. To do so, you will have to prove and use the identity,

$$\int_0^1 x^n (1 - x)^m dx = \frac{n!m!}{(n + m + 1)!}.$$

[Hint: To prove the identity, let $C(n, m)$ be the said integral. Integrate by parts to prove that

$$C(n, m) = \frac{m}{n + 1} C(n + 1, m - 1).$$

Then, start with $C(n, 0) = 1/(n + 1)$, and derive the identity by induction on m .]