Reading and Problem Assignment #1
Math 501–1, Spring 2006
University of Utah

Read chapter 1 (combinatorial analysis). Skip the starred sections.

The following are borrowed from your text.

Problems:

1. Twenty workers are to be assigned to 20 different jobs, one to each job. How many different assignments are possible?

2. Consider a group of 20 people. If everyone shakes hands with everyone else, then how many handshakes take place?

3. Five separate awards (best scholarship, best leadership qualities, and so on) are to be presented to selected students from a class of 30. How many different outcomes are possible if:
(a) a student can receive any number of awards;
(b) each student can receive at most 1 award?

4. A person has 8 friends, of whom 5 will be invited to a party.
(a) How many choices are there if 2 of the friends are feuding and will not attend together?
(b) How many choices if 2 of the friends will only attend together?

Theoretical Problems:

1. Verify that $\binom{n}{k} = \binom{n}{n-k}$. Use this to prove that

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2.$$

[Hint for the second part: In order to choose n from $2n$ people, you must choose some number $k \in \{0, \cdots, n\}$ from the first n people.]