

## Announcements

Our final examination will take place in LS 107, on Wednesday, May 2, from 4-6 pm.

### Arithmetic Rules for the Logarithm

1.  $\log_a 1 = 0$ .
2.  $\log_a a = 1$ .
3.  $\log_a(MN) = \log_a M + \log_a N$ .
4.  $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$ .
5.  $\log_a(M^p) = p \log_a(M)$ .
6.  $\log_a M = \frac{\log_b M}{\log_b a}$ .
7.  $x = \log_a(a^x) = a^{\log_a x}$ .

### Problems

For each of the following problems (except the first two), justify each of your manipulations involving logarithms using the above rules. Do not use calculators for these questions.

1. Prove that  $\log_a\left(\frac{M}{N}\right) = \log_a(M) - \log_a(N)$ .
2. Prove that  $\log_a(M^p) = p \log_a(M)$ .
3. If  $\log_4 7 = n$ , find  $\log_4 \frac{1}{7}$ .
4. Find the base  $b$  for which  $\log_b 16 = \log_6 36$ .
5. Write  $\frac{1}{2} = \log_9 3$  in exponential form.
6. Compute  $\ln 50067400 - \ln 500674 - 2 \ln 10$ .
7. Sketch the graphs of  $f(x) = a^x$  and  $g(x) = b^x$ , given that  $b > a > 1$ , on the same axes. Comment specifically on how the two graphs compare, namely, where they intersect, which one lies above the other on what set, etc.
8. If  $f$  is an exponential function, express  $f(-x)$  in terms of  $f(x)$ .

## Problem Set 6

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9. (The Magic Penny and the US national debt) Suppose you have a magic penny that “doubles” into two magic pennies every night when kept underneath your pillow as you sleep. Look up the size of the current US national debt. Calculate how many nights it would take for magic pennies to “double” to the current US national debt, starting with one magic penny. (Of course, for this question, ignore your pillow size, the frequency of your sleepless nights, etc.)
  10. Textbook problems: 5.1.29, 5.1.31.
  11. Textbook problems: 5.2.54, 5.2.56.
  12. Textbook problems: 5.3.2, 5.3.5, 5.3.8, 5.3.16, 5.3.18, 5.3.28, 5.3.35.