Linear Inequalities in Two Variables

Consider the linear function $f(x, y) = ax + by + c$, in two variables x and y, where $a, b, c \in \mathbb{R}$ are constants.

Recall that the graph Graph(f) of f is a plane in 3 dimensional space.

Diagram 1

Question Does the plane Graph(f) always intersect the xy plane?

Answer _____.

Graph(f) intersects the xy plane only when it is not parallel to the xy plane. The fact that Graph(f) is parallel the xy plane is equivalent to f being a

_____,

or equivalently,

$$a = b = \_\_\_\_\_.$$

Question How does the plane Graph(f) intersect the xy plane?

Answer If f is not constant[1], then Graph(f) intersects the xy plane in a

_____.

Question How can the straight line of intersection of Graph(f) with the xy plane be described?

Answer

$\{ \quad (x, y) \in \mathbb{R}^2 \quad | \quad$ _____ $\}$, or

$\{ \quad (x, y) \in \mathbb{R}^2 \quad | \quad$ _____ $\}$.

[1] equivalently Graph(f) is not parallel to the xy plane; equivalently, a and b are not both zero.

Observation The line $l : f(x, y) = ax + by + c = 0$ divides the xy plane into two halves.

Diagram 2

Question How does the value of f compare with 0 on each half plane?

Answer

_____ on one of the half plane, and

_____ on the other.

Question How does the value of f change as you move in a direction perpendicular to $l : f(x, y) = ax + by + c = 0$?

Diagram 3

Answer

f keeps _____ as we move in the direction of "$f > 0$".

f keeps _____ as in the opposite direction.