Review Problems for Final Examination

1. Simplify $\dfrac{\frac{t^2}{\sqrt{t^2+1}} - \sqrt{t^2+1}}{t^2}$

2. Solve the following equations.

   (a) $16x^3 + 56x^2 + 49x = 0$

   (b) $4x^2 - 4x + 4 = 0$

3. Plot the points $(2, 4)$ and $(4, -4)$, and find the slope of the line passing through them.

4. Find an equation of the line passing through the points $(-1, 4)$ and $(6, 4)$.

5. Find an equation of the line passing through the point $(0, 1)$ that is parallel to the line passing through the points $(3, 6)$ and $(-6, 0)$.

6. Sketch the graph of the line passing through the point $(0, 10)$ with slope $m = -1$.

7. Your salary was \$28,500 in 1994 and \$32,900 in 1996. If your salary grows linearly, what will your salary be in 2001?

8. What is a function?

9. Determine if the equation represents y as a function of x. If not, explain why.

   (a) $x^2 + y^2 = 4$

   (b) $|y| = 4 - x$

   (c) $x^2 + y = 4$

10. Find the domain of the following functions.

    (a)
    $$f(x) = |4 - x|$$

    (b)
    $$f(x) = \sqrt{x^2 - 5}$$

    (c)
    $$f(x) = \frac{1}{x^2 - 1}$$

11. Evaluate the function at the specified values of the independent variable and simplify.

    (a) $f(x) = \sqrt{x + 8} + 2$ at $x = -8$

    (b) $g(y) = 7 - 3y$ at $y = s + 2$

    (c) $f(x) = \begin{cases} x^2 + 2 & x \le -2 \\ x + 2 & x > -2 \end{cases}$ at $x = 2$, $x = -4$, and $x = -2$

12. Sketch the graph of $f(x) = (x + 2)^3 + 1$.

13. Let $y = f(x)$ be a function. How are the graphs of $f(x)$ and $f(-x)$ related? $f(x)$ and $-f(x)$?

14. Find $f \circ g$, $g \circ f$ and $f \circ f$. Simplify your answer.

    (a) $f(x) = \sqrt[3]{x - 1}$, $g(x) = x^3 + 1$

    (b) $f(x) = x^2$, $g(x) = x + 1$

15. If f and g are inverse functions, what are $(f \circ g)(x)$ and $(g \circ f)(x)$?

16. Without solving for the inverse function, does $q(x) = (x - 5)^2$ have an inverse?

17. Determine if the following functions have an inverse and if it does, find it.

    (a)
    $$f(x) = \frac{3x + 4}{5}$$

    (b)
    $$f(x) = \sqrt{x - 2}$$

18. A triangular sign has a height that is twice its base. The area of the sign is 10 square feet. Find the base and height of the sign.

19. What is the inverse of $f(x) = \ln(x)$? What about $g(x) = 10^x$?

20. Write the following quadratic polynomials in standard form, $y = a(x - h)^2 + k$, identify the vertex, and graph the function.

    (a) $f(x) = (x + 5)^2 - 6$

    (b) $h(x) = x^2 + 2x + 1$

    (c) $q(x) = -x^2 - 4x + 1$

21. Write an equation for a quadratic function whose graph opens down and has the following x-intercepts: $(4, 0)$ and $(8, 0)$. (Note that the answer is not unique.)

22. The profit for a company is given by $P(x) = -0.0002x^2 + 140x - 250,000$, where x is the number of units sold. What sales level will yield a maximum profit?

23. Divide

    (a) $\dfrac{x^3 + x^2 - 64x - 64}{x + 8}$

    (b) $\dfrac{2x^3 + 3x^2 - 3x - 2}{x - 2}$

24. Factor the following polynomials over the complex numbers.
    (a) $h(x) = 3x^3 - 4x^2 + 8x + 8$ given that $x = 1 - \sqrt{2}i$ is a root.
    (b) $f(y) = y^4 - 625$

25. Find a polynomial function with integer coefficients that has the following zeros: $-5, -5, 1 + i\sqrt{3}$.

26. Write the following in the standard form $a + bi$.
    (a) $\dfrac{(2-3i)(5i)}{2+3i}$
    (b) $\dfrac{1+i}{i} - \dfrac{3}{4-i}$

27. Find all asymptotes for the rational functions.
    (a) $h(x) = \dfrac{x^2}{x^2+9}$
    (b) $p(x) = \dfrac{2}{x^3 - 2x^2}$
    (c) $g(x) = \dfrac{x^3}{2x^2 - 8}$

28. Find a rational function that has vertical asymptotes at $x = 3$ and $x = -2$.

29. Find a rational function that has a vertical asymptote at $x = 3$ and a slant asymptote $y = x$.

30. Find a rational function that has its only vertical asymptote at $x = 0$, a horizontal asymptote at $y = 1/3$, and intersects the x-axis at $x = 3$ and $x = -2$.

31. Find the partial fraction decomposition.
    (a) $\dfrac{x}{(x^2 + 2x + 1)}$
    (b) $\dfrac{x+2}{x^2 - 4x}$
    (c) $\dfrac{5x+2}{(x+1)(x^2 - 2x + 3)}$

32. Identify all asymptotes of $y = ae^{x-b} + c$.

33. Identify all asymptotes of $y = a\ln(x - b) + c$.

34. Sketch the graphs.
    (a) $y = e^x$
    (b) $y = \ln(x)$
    (c) $y = e^{-x}$
    (d) $y = e^{x-2} + 3$
    (e) $y = -\ln(x)$
    (f) $y = \log(x + 3)$

35. Write as one logarithm.
$$\ln(x) + 2\ln(y+1) - \frac{1}{2}\ln(z)$$

36. Change to base 10 and base e.
$$\log_2(x+1)$$

37. Solve the equations for x.
    (a)
$$3(5^x) - 2 = 20$$
    (b)
$$e^x + 10e^{-x} = 7$$
    (c)
$$\log(x-1) = \log(x-2) - \log(x+2)$$
    (d)
$$\frac{100}{1 + \ln(x)} = 10$$

38. Suppose you invest \$1000 in the stock market. After two years the value of your investment is \$1200. Suppose that the growth rate is constant and the growth is exponential. What will be the value in t years? How long will it take your investment to triple? Suppose you invested \$10,000 instead of \$1000, how long will it take your investment to triple?

39. Find all points where the graphs intersect.
$$y = x^3 + 3x + 2$$
$$y = x^2 + 4x + 2$$

40. Solve the system by any method that works.
    (a)
$$\begin{aligned} 13x &+ 31y &&= -8 \\ -6x &- 19y &&= -1 \end{aligned}$$
    (b)
$$\begin{aligned} 2x &+ 2y &- 6z &= 5 \\ -3x &+ y &- z &= -2 \\ -x &- y &+ 3z &= 4 \end{aligned}$$

(c)

$$\begin{array}{rcrcrcr} 5x & - & 2y & + & 3z & = & -9 \\ 4x & + & 3y & + & 5z & = & 4 \\ 2x & + & 4y & - & 2z & = & 14 \end{array}$$

(d)

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 10 \\ -x & + & y & + & z & = & 4 \end{array}$$

41. Compute the inverse of the matrix. How can you check your answer?

$$\begin{bmatrix} 2 & -3 & 2 \\ 1 & 2 & 1 \\ 0 & 2 & -1 \end{bmatrix}$$

42. Use the inverse you found in the previous problem to solve.

$$\begin{array}{rcrcrcr} 2x & - & 3y & + & 2z & = & -1 \\ x & + & 2y & + & z & = & 3 \\ & & 2y & - & z & = & 5 \end{array}$$

43. Suppose that A,B,C,D are matrices. A is 2×2; B is 2×3; C is 2×1; D is 2×1. Which of the following operations are defined?

(a) $2B$
(b) $A + B$
(c) $AC + D$
(d) D^{-1}
(e) $|C|$
(f) AB
(g) $|A|$
(h) BA

44. Compute the determinant.

$$\begin{vmatrix} 5 & 5 & -1 & 2 \\ 0 & 7 & 4 & 3 \\ 0 & 2 & 0 & 0 \\ -1 & 4 & 2 & 0 \end{vmatrix}$$

45. What does a zero determinant tell you about a matrix? What does it tell you about the equation $AX = B$?

46. Create a system of three linear equations with three unknowns which has infinitely many solutions.

47. Create a system of three linear equations with three unknowns which has no solution.

48. The distance from your house to Wendover is 120 miles. Because of traffic and you drive at two different speeds, 60 miles per hour in traffic and 80 miles per hour when you are all alone. Suppose that your last trip took 1 hour and 40 minutes. How much of the time were you traveling 60 mph, and how long were you traveling 80 mph?

49. Use sigma notation to write the sum.

$$1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \dots$$

50. Write an expression for the n^{th} term, a_n, of the sequence. Assume that the first term is a_1. Identify the sequences as arithmetic, geometric, or neither.

(a)

$$1,\ -\frac{2}{3},\ \frac{4}{9},\ -\frac{8}{27}, \dots$$

(b)

$$-2,\ 3,\ 8,\ 13, \dots$$

(c)

$$1,\ \frac{x^2}{2!},\ \frac{x^4}{4!},\ \frac{x^6}{6!}, \dots$$

51. Find a formula for a_n for the arithmetic sequence with $a_6 = 30$ and $a_{11} = 64$.

52. Find the sum of the first 1000 positive odd integers.

53. Find a formula for the n^{th} term in the geometric sequence if the fourth term is $\frac{2}{27}$ and the sixth term is $\frac{2}{243}$. Note that there are two possible answers.

54. You deposit \$100 at the beginning of each month into an account with an annual percentage rate of 4% compounded monthly. The value of each deposit plus interest after 60 months defines a sequence, where d_n is the value of the deposit made n months ago. For example,

$$d_1 = 100\left(1 + \frac{0.04}{12}\right)^1$$

$$d_2 = 100\left(1 + \frac{0.04}{12}\right)^2$$

$$d_n = 100\left(1 + \frac{0.04}{12}\right)^n \quad \text{for } n \le 60.$$

Find the balance of the account by computing

$$\sum_{k=1}^{60} d_k$$

55. Compute the sum.

$$\sum_{k=0}^{\infty} \left(\frac{5}{9}\right)^k$$

56. Expand $(x^2-2)^5$ using the binomial theorem or Pascal's triangle.

57. Expand and simplify $(1-i)^8$

58. Find the coefficient of x^6y^5 in the expansion of $(2x^2-3y)^8$.